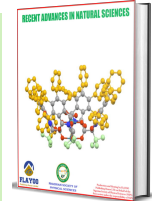



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Modelling Patients Waiting and Service Time by ARIMA Model: A Case of Federal University Gusau Clinic

Aliyu Usman Moyi, Kabir Bello, Olayemi Joshua Ibidoja , Garba Muhammad

Department of Mathematics, Federal University Gusau, Nigeria

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ABSTRACT

The ability to model and forecast waiting and service time to increase patients' satisfaction, reduce waiting time, avoid casualties, and increase efficiency in service delivery is crucial. It encourages the identification of future pressure by using the relevant key performance indicators. In this paper, the ARIMA model is used to study the waiting and service time of patients at the *Federal University Gusau Health Services Clinic*. The system was a single, time-independent arrival with many service points. Based on the results found in the waiting and service processes, the service time has a lower mean and variance when compared to the waiting time. The waiting time has a lower skewness and kurtosis when compared to the service time. The Ljung-Box (Q) Statistic test shows that the correlation in the time series has been adequately captured for the waiting and service time processes, though the waiting and service time processes have 4 and 10 outliers respectively. The ARIMA (0,1,2) and ARIMA (2,1,1) are selected for modelling the waiting and service time respectively based on the evaluation metrics.

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1. INTRODUCTION


The use of modelling and forecasting methods in healthcare settings can lead to operational improvements and improved patient care, and modifying a time series to a common day clustered sequence can provide a statistically significant improvement in the accuracy of a forecast [1]. Modelling and forecasting outpatient visits is a vital task in the management of any healthcare organization. Accurate patient forecasts can lead to improvements in staff scheduling, resource utilization, and reduced wait times. Furthermore, it can also provide administrators with a quantified

basis for decision making [2]. The use of modelling methods in healthcare settings can ultimately lead to operational improvements, which directly translate to better patient care. Time series forecasting is widely used in industries such as manufacturing, distribution, and electric utilities [3].

Accurate modelling and forecasting of hospital outpatient medical needs is beneficial for reasonable planning and allocation of healthcare resources to meet the medical demands. In terms of the multiple attributes of daily outpatient visits, such as randomness, cyclicity and trend, time series methods such as ARIMA can be a good choice for outpatient visit forecasting [4].

It has been shown that incorporating modelling and forecasting methods in a healthcare setting can improve resource management and hence the overall efficiency of the system [5]. For this

*Corresponding author: Tel.: +234-703-2825-699

e-mail: ojibidoja@fugusau.edu.ng (Olayemi Joshua Ibidoja )

reason, existing work on modelling and forecasting in healthcare focuses on problems such as predicting patient demand, resource scheduling and bed assignments.

Extensive research has been conducted on time series modelling and forecasting across a variety of industries. Exponential smoothing methods were applied to forecasting electricity demand lead times from a half-hour to a day-ahead [6]. The forecast of the physical cash needs of European banks and found the autoregressive integrated moving average (ARIMA) method to be optimal studies [7]. The insight on building a distribution model for use in modelling and forecasting a time series where demand periodically drops to zero [8].

In the articles related to modelling and forecasting in healthcare settings outside of the emergency department (ED). The new hybrid models for time series forecasting were investigated and ARIMA where combined with other methods [9, 10]. The exploration of the use of calendar and weather variables, and stepwise linear regression to forecast patient visits to an urgent care centre were examined [11]. Another study investigated forecasts from the perspective of required nurse staffing levels [5]. Several researchers have examined the importance of forecasting daily patient volumes in hospital EDs. In another study, it focused on 1 to 30 day-ahead forecasts using ARIMA, regression, and neural network techniques [12]. The analysis of patient arrivals as well as length of stay and focused on forecasting using various moving average techniques were studied [13, 14].

In this study, a step was taken further to explore whether other factors affect the length of stay in the ED and found that an autocorrelation exists between the mean length of stay of the current day and the previous day [15]. ARIMA models for forecasting daily attendances at the ED was useful, and readily available tool for predicting ED workload [16, 17]. The ARIMA model was used to forecast monthly outpatient visits in a general hospital in China [18]. In another study, hospital daily outpatient visits forecasting using a combinatorial model based on ARIMA and single exponential smoothing models for fitting and forecasting performances in two departments for 8 weeks, it was found that the ARIMA model for the respiratory outpatient visits department had 15.97% performance and the endocrinology outpatient visits department had 23.48% performance [4]. The ARIMA model was used to analyse the typhoid mortality rate in Delta State, Nigeria, and the ARIMA (0, 1, 0) model was the best model for forecasting [19].

The Clinic at *Federal University Gusau Nigeria (FUGN)*, is the only one in the University since the establishment of the University in 2013. The Clinic does not engage in any modelling and forecasting research that can solve the problem of staff scheduling and reduce patient waiting time. The Clinic is currently experiencing an increasing number of patients due to the number of students and staff.

The contribution of the study is to reduce patients waiting time for service delivery. Hence, a time series forecasting model will be used to analyse the patient waiting time and service time using the ARIMA model. Forecast values for the waiting and service time for the patients of *FUGN* clinic are needed for better and more effective managerial decisions to improve clinic efficiency.



Fig. 1. Patient waiting for service



Fig. 2. The research methodology.

2. MATERIALS AND METHODS

The methods applied in this study are stated in this section. Figure 1 shows the representation of the patients waiting to be served. Figure 2 shows the summary of research methodology. The waiting and service time are collected, exploratory data analysis is carried out before the ARIMA model.

3. DATA DESCRIPTION AND COLLECTION

This research is based on the waiting and service time of patients that arrive between 8:00 am to 2:00 pm at the *FUGN* Clinic. The primary data were collected from the Clinic using 537 patients, and collected between 25 June, 2019 to 23 July, 2019. The system was treated as single, with time-independent arrivals with numerous service points [20, 21]. The arrival time, waiting time and service time were collected using observation method.

4. AUTOCORRELATION AND PARTIAL AUTOCORRELATION

The initial correlation of the observations in a time series is usually expressed as a function of the time lag between observations. The partial autocorrelation measures the correlation between the observations a particular number of time units apart in a time series, after controlling for the effects of observations at intermediate time point [22]. The autocorrelation at lag k , $\gamma(k)$, is defined mathematically as [23, 24].

$$\gamma(k) = \frac{E(X_t - \mu)(X_{t+k} - \mu)}{E(X_t - \mu)^2}, \quad (1)$$

where $X_t, t = 0, \pm 1, \pm 2, \pm 3, \dots$ represent the values of the series and μ is the mean of the series. E denotes the expected value, the corresponding sample statistic is calculated as follows [23–25]:

$$\widehat{\gamma}(k) = \frac{\sum_{i=1}^{n-k} (x_i - \bar{x})(x_{i+k} - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}, \quad (2)$$

where \bar{x} is the mean of the series of observed values x_1, x_2, \dots, x_n . A plot of the sample values of the autocorrelation against the lag is known as the autocorrelation function or correlogram and is a basic tool in the analysis of time series particularly for indicating possibly suitable models for the series. The term in the numerator of $\gamma(k)$ is the autocovariance. A plot of the autocovariance against lag is called the autocovariance function [26].

5. ARIMA (P, D, Q)

ARIMA models are in theory the most general class of models for forecasting a time series, which can be made to be “stationary” by differencing [27]. Perhaps in conjunction with nonlinear transformations such as logging or deflating [28]. A random variable that is a time series is stationary if its statistical properties are all constant over time [29]. The time series modeler procedure which creates models for time series and produces forecasts in SPSS was used to analyze the data. It includes an expert modeler that automatically determines the best model for each of your time series. For experienced analysts who desire a greater degree of control, it also provides tools for custom and select the best ARIMA model.

6. AUTOREGRESSIVE MODEL

This is a model used primarily in the analysis of time series in which the observations z_t at time t function of previous values of the series. The model is represented below [24].

$$z_t = \Phi_1 z_{t-1} + \Phi_2 z_{t-2} + \dots + \Phi_p z_{t-p} + a_t, \tag{3}$$

where a_t is the random disturbance and $\Phi_1, \Phi_2, \dots, \Phi_p$ are finite set of weight parameters. The process above is the autoregressive process of order p . The autoregressive process of first order ($p = 1$) and second order ($p = 2$) are represented respectively in Eqs. (4) and (5) [24].

$$z_t = \Phi_1 z_{t-1} + a_t. \tag{4}$$

$$z_t = \Phi_1 z_{t-1} + \Phi_2 z_{t-2} + a_t. \tag{5}$$

7. MOVING AVERAGE PROCESS

The moving average process is represented as follows [24]:

$$z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q}, \tag{6}$$

where $a_t, a_{t-1}, \dots, a_{t-q}$ are white noise sequence, $\theta_1, \theta_2, \dots, \theta_q$ are finite set of weight parameters. The process above is called moving average of order q . The moving average process of first order ($q = 1$) and second order ($q = 2$) are represented respectively in Eqs. (7) and (8) [24]:

$$z_t = a_t - \theta_1 a_{t-1}. \tag{7}$$

$$z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}. \tag{8}$$

8. AUTOREGRESSIVE MOVING AVERAGE MODEL

A model for a time series that combines both an autoregressive model and a moving-average model. The general model of order p, q usually denoted by ARMA(p, d, q) is shown below [24]:

$$x_t = \Phi_1 x_{t-1} + \Phi_2 x_{t-2} + \dots + \Phi_p x_{t-p} + a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q}, \tag{9}$$

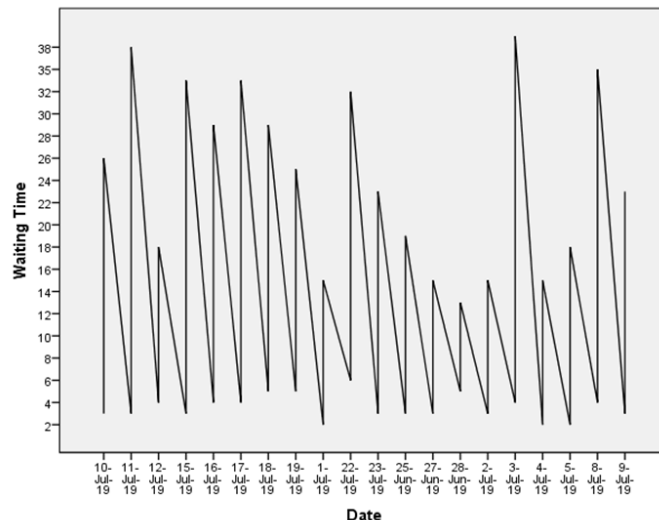


Fig. 3. Plot of the Distribution of Waiting Time by Patients at the Clinic.

where $\Phi_1, \Phi_2, \dots, \Phi_p$ and $\theta_1, \theta_2, \dots, \theta_q$ are the parameters of the model and $a_t, a_{t-1}, \dots, a_{t-q}$ are a white noise sequence. In some cases, such models are applied to the time series observations after differencing to achieve stationarity, in which case they are known as autoregressive integrated moving-average models. The p is the number of autoregressive terms, d is the number of no seasonal differences needed for stationary, and q is the number of lagged forecast errors in the prediction equation.

9. LJUNG-BOX TEST

The test is used to test whether there is a collection of autocorrelations of a time series is less than 0. The test statistic is shown below [30]:

$$Q = T(T + 2) \sum_{k=1}^s \frac{r_k^2}{T - K}, \tag{10}$$

where T is the number of observations, s is the coefficient length for autocorrelation test, r_k is the autocorrelation coefficient for lag k .

The hypothesis of Ljung-Box test are:

H_0 : Residual is white noise.

H_a : Residual is not white noise.

If the sample value of Q is greater than the critical value of the χ^2 distribution with s degrees of freedom, then at least a value of r is statistically different from zero at the specified level of significance.

10. RESULTS AND DISCUSSION

The model assessed could capture the trend of the waiting and service time, it results in the estimation of the different trend to each waiting and service time.

Table 1 shows the basic summary statistics of the waiting and service time of the patients. The service time has a lower mean and variance when compared to waiting time. This means that the patients spent more waiting time than service time and the variability among the service time for patients is lower when compared to the waiting time for patients. The mean is sensitive to

Table 1. Basic Statistics.

	Mean		Variance		Skewness		Kurtosis	
	Statistic	Std. Error	Statistic	Statistic	Std.Error	Statistic	Std.Error	
Waiting Time	11.54	0.294	46.361	1.329	0.105	2.235	0.210	
Service Time	5.50	0.130	9.039	6.904	0.105	90.954	0.210	

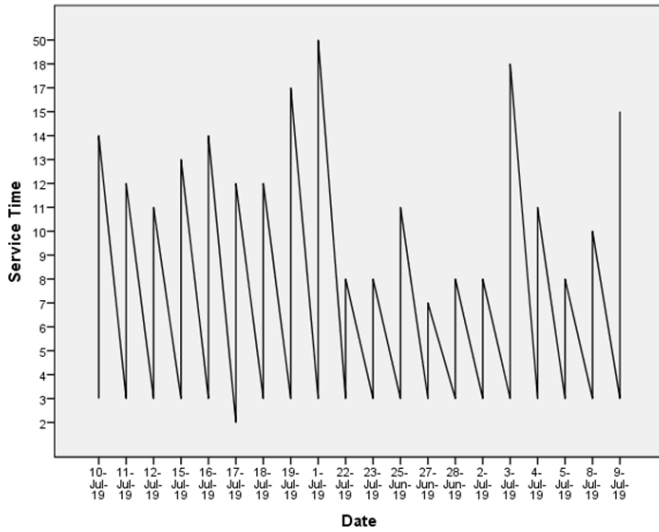


Fig. 4. Plot of the distribution of Service Time by Patients at the Clinic.

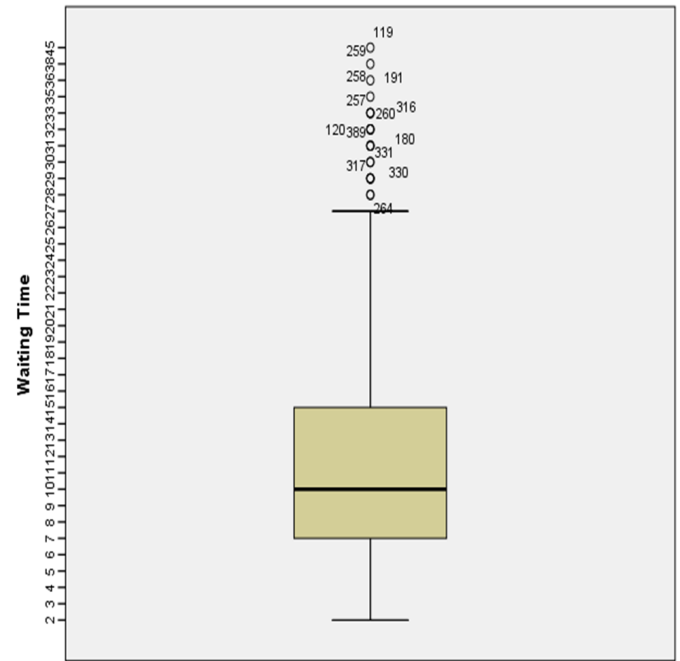


Fig. 6. Box Plot of the Distribution of Waiting Time by patients at the Clinic.

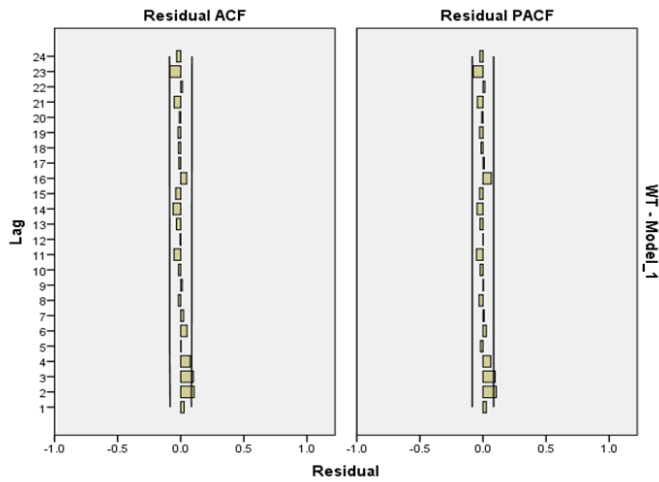


Fig. 5. Correlogram Waiting Time by Patients at the Clinic.

Table 2. Model Description.

Model	Model Type
Waiting Time	ARIMA (0,1,2)
Service Time	ARIMA (2,1,1)

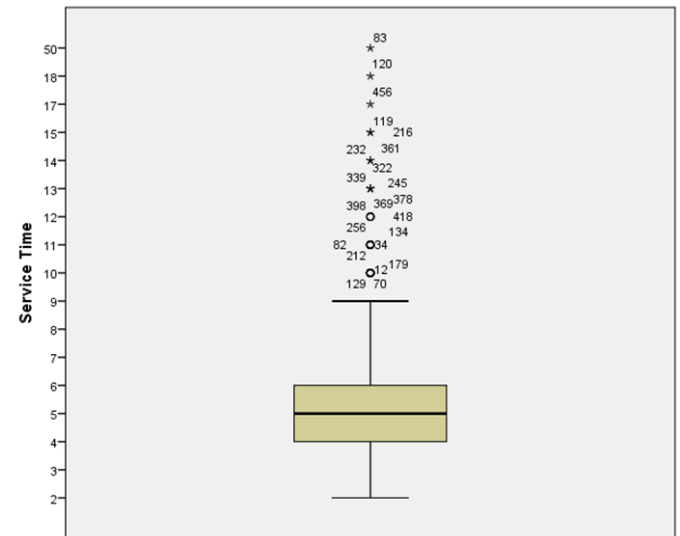


Fig. 7. Box Plot of the Distribution of Service Time by patients at the Clinic.

outliers and observations that depart from the distribution’s general form or shape are called outliers [31]. The waiting time has a lower skewness and kurtosis when compared to service time. Skewness is a measure of the degree and direction of asymmetry. Kurtosis is used to measure the extreme tail for outliers. Table 2 shows the ARIMA model for the waiting and service time.

The time series plots in Figures 3 and 4 are non-stationary series, the non-stationary series was supported by the low stationary *R* squared values in Tables 3, 4 and 5. To make the time series stationary, hence, the condition that difference $d = 1$ was used and transform the data using first order differencing to make it

Table 3. Model Summary for the Waiting Time.

Fit Statistic	Mean	Percentile						
		5	10	25	50	75	90	95
Stationary R-squared	0.417	0.417	0.417	0.417	0.417	0.417	0.417	0.417
R-squared	0.293	0.293	0.293	0.293	0.293	0.293	0.293	0.293
RMSE	5.764	5.764	5.764	5.764	5.764	5.764	5.764	5.764
MAPE	53.47	53.47	53.47	53.47	53.47	53.47	53.47	53.47
MaxAPE	370.8	370.8	370.8	370.8	370.8	370.8	370.8	370.8
MAE	4.462	4.462	4.462	4.462	4.462	4.462	4.462	4.462
MaxAE	23.18	23.18	23.18	23.18	23.18	23.18	23.18	23.18
Normalized BIC	3.585	3.585	3.585	3.585	3.585	3.585	3.585	3.585

MAPE is Mean Absolute Percentage Error.

MSE is Mean Squared Error.

RMSE is Root Mean Squared Error.

Table 4. Model Summary for the Service Time.

Fit Statistic	Mean	Percentile						
		5	10	25	50	75	90	95
Stationary R-squared	0.6300	0.6300	0.6300	0.6300	0.6300	0.6300	0.6300	0.6300
R-squared	0.31700	0.31700	0.31700	0.31700	0.3170	0.31700	0.31700	0.31700
RMSE	1.94700	1.94700	1.94700	1.94700	1.9470	1.94700	1.94700	1.94700
MAPE	30.2580	30.2580	30.2580	30.2580	30.2580	30.2580	30.2580	30.2580
MaxAPE	148.498	148.498	148.498	148.498	148.498	148.498	148.498	148.498
MAE	1.46700	1.4670	1.46700	1.46700	1.46700	1.46700	1.46700	1.46700
MaxAE	7.65300	7.6530	7.65300	7.65300	7.65300	7.65300	7.65300	7.65300
Normalized BIC	1.49700	1.4970	1.49700	1.49700	1.49700	1.49700	1.49700	1.49700

Table 5. Model Statistics.

Model	Number of Predictors	Stationary R-squared	Ljung-Box Q (18)			Number of Outliers
			Statistics	DF	Sig.	
Waiting Time	0	0.417	23.421	16	0.103	4
Service Time	0	0.630	14.659	15	0.476	10

stationary. ARIMA (0, 1, 2) for the waiting time model, with stationary R-Square of 0.417 and non-significance of Q-Statistic, which gives all significance model parameter without constant is appropriate for forecasting Waiting Time of patients at FUGN Clinic. Figure 5 shows the spikes of autocorrelation and partial autocorrelation function obtained from the ARIMA (0, 1, 2) for waiting time, the model is not statistically significant. This implies that the model is reasonable fit to the waiting time data. The variability of the waiting and service time are shown in Figures 6 and 7. The points outside the box-plot are the outliers. A box-plot uses the 5-number summary of Q1, Q2, Q3, minimum and maximum value to summarize the data.

After building series of ARIMA models, it was found that ARIMA (2, 1, 1) for the service time model, with Stationary R-Square of 0.630 and non-significance of Q-Statistic indicate better ARIMA model in forecasting service time at FUGN Clinic, all model parameters are said to be significance except the constant.

Different evaluation metrics such as MAPE, MSE, RMSE etc. have been used for model assessment [32]. Table 3 shows the different evaluation metrics for the waiting time, the stationary

R-squared is higher than R-squared. The RMSE and MAPE are 5.564 and 53.47. Generally, a lower value of RMSE and MAPE are good for predictions. All the validation measures such as RMSE, MAPE, and MAE indicate that significant results are obtained by the waiting time of patients.

The value MAPE (53.47) signifies the average percentage error between the waiting time of patients predicted by the model and the real value. If an MAPE value is small, it is high prediction accuracy [32]. The value R-square (0.9732), implies the percentage of the variance explained by the model. Furthermore, it also gives the value of the variables at different percentiles and the distribution of the variables. Column 25 is the 25% percentile, which is the first quartile, column 50 is the 50% percentile, which is the median, the median is less sensitive to extreme observations, column 75 is the 75% percentile, which is the third quartile.

Table 5 shows the Ljung-Box (Q) Statistic test was performed using SPSS. The Ljung-Box Statistic of the model for the waiting time of patients is not significantly different from zero, with a value of 23.421 for 16 d.f., with 10 outliers and associated p-value of 0.103, hence, it fails to reject the null hypothesis of white

noise. This implies that the correlation in the time series has been adequately captured. Similarly, the Ljung-Box (Q) Statistic for the model for the service time of patients is not significantly different from zero, with a value of 14.659 for 15 d.f., with 4 outliers and associated p-value of 0.476, hence, it fails to reject the null hypothesis of white noise. This implies that the correlation in the time series has been adequately captured.

11. CONCLUSION AND FUTURE WORK

In this work, an ARIMA model is proposed to identify waiting and service characteristics, modelling and predictions. The ARIMA (0,1,2) and ARIMA (2,1,1) are selected for modelling the waiting and service time respectively based on the evaluation metrics. It was observed from the results that the service time has more outliers than the waiting time. This could be due to the fact that the time required to attend to a patient by a doctor depends on the patient and the nature of the sickness. This is also evident in the graphs, where the series exhibit non-stationary. Despite the nature of the data and results obtained, the accuracy of the ARIMA model prediction is suitable and satisfactory.

For future studies, we want to build on the results of this study. A hybrid model using machine learning algorithms and statistical methods can be applied to solve the problem.

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