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Integrative approaches of multivariate convolutional neural networks with a dynamic conditional correlation model for forecasting stock market volatility

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ABSTRACT

Machine learning and deep learning approaches have been extensively studied across various fields, significantly contributing to the successful resolution of numerous problems. Examples include image classification, computer vision applications, and natural language processing. Owing to their powerful capabilities, they have not only made significant contributions in the fields above but also increasingly been applied to time series analysis, particularly in analyzing financial time series to forecast the volatility and correlation among stock markets. Researchers have proposed hybrid models that combine machine learning and deep learning approaches with statistical and econometric models to enhance time series analysis and forecasting. In this work, the dynamic conditional correlation (DCC) model, as an econometric model, and multivariate convolutional neural networks (MCNNs), as deep learning models, are employed to construct hybrid models to enhance the forecasts of dynamic volatility and correlation among stock markets. Two methods are presented for combining the DCC model with the convolutional neural network (CNN) model. The results show that the best way to combine the DCC model and the CNN model is to use the outputs of the CNN model as inputs to the DCC model. The hybrid DCC-MCNN model demonstrates stronger performance across both in-sample and out-of-sample accuracy measures, such as the root mean square error (RMSE) and mean absolute error (MAE). Specifically, the hybrid DCC-MCNN model emerges as the top performer among the evaluated models, surpassing both the single model approaches and the hybrid MCNN-DCC model in forecasting time-varying volatility and correlation among stock markets.

1. INTRODUCTION

It is widely recognized that accurate analysis and prediction of temporal relationships within data are challenging because of the presence of nonlinear trends, heavy-tailed distributions, and inherent noise [1]. These characteristics are particularly prevalent in financial time series, especially in stock market data. When

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models for forecasting financial data are developed, it is essential that they can learn nonlinear dependencies within the data and exhibit high resistance to noise. Traditional time series models such as VAR and VARMA are inadequate for capturing these nonlinear patterns. Furthermore, financial time-series models, including GARCH, EGARCH, and EWMA, provide both advantages and limitations in forecasting stock market volatility. These models exploit various characteristics, including leverage effects, excess kurtosis, and volatility clustering, which are commonly observed in financial time-series data [2].

In recent years, machine learning and deep learning [3] approaches have been extensively studied across various fields, significantly contributing to the successful resolution of numerous problems. For instance, Ref. [4] employed random forest for classification prediction modeling, Ref. [5] utilized support vector machine and random forest for image classification. Convolutional neural networks (CNNs) [6, 7] have demonstrated outstanding performance in computer vision applications. Similarly, recurrent neural network (RNN) models have gained increasing importance in natural language processing (NLP). Owing to their powerful capabilities, they have not only made significant contributions in the aforementioned fields but have also increasingly been utilized in time series analysis [8, 9]. They have been employed for univariate [10] and multivariate time series [11–14]. They have also been used for different purposes such as anomaly detection in time series [15], classification of audio signals [16], and feature extraction for human activity recognition [13]. In addition, they have been applied to different time series data and prediction tasks [17] such as weather prediction [18], forecasting energy [19, 20], and forecasting crude oil prices [21]. They have also been used extensively in financial time series analysis, including forecasting cryptocurrency volatility [22], forex exchange [23] , stock prices [24], and stock market returns.

For efficient use of machine learning and deep learning in time series analysis and forecasting, particularly in financial time series, researchers have proposed hybrid models combining machine learning algorithms with deep learning approaches. Other combinations include machine learning with statistical/econometric models, as well as deep learning approaches integrated with statistical/econometric methods. Many scholars have studied the application of machine learning methods, deep learning approaches, and their integration with statistical and econometric models in time series modeling and forecasting. Ref. [25] offers a review of the advancements in deep learning and unsupervised feature learning for time series issues in general, while Ref. [26] provides an up-to-date review of machine learning applications in financial time series forecasting. The review of Ref. [27] provides a thorough literature review of deep learning studies on financial time series forecasting implementation. More specifically, Ref. [28] surveyed the use of deep neural networks in forecasting the stock market, highlighting needs, challenges, and future directions. Ref. [29] examined hybrid structures by reviewing over 150 papers that utilized various hybrid models in time series modeling and forecasting domains, while Ref. [30] provided an experimental review on deep learning architectures that used for time series forecasting.

Ref. [31] used gradient descent boosting, random forest, support vector machine, and artificial neural networks to improve the accuracy of volatility forecasts made by hybrid models on the basis of combinations of GARCH-type models and [32] constructed a hybrid model by integrating linear regression model with deep belief networks for time series prediction and the results showed that the hybrid model exhibited high accuracy in forecasting time series.

Ref. [33] developed a system using deep convolutional neural networks (CNN) and novel planar feature representation methods to enhance the algorithmic trading frameworks. The system is implemented and evaluated via historical datasets of the Taiwan Stock Index Futures. The experimental results demonstrate the effectiveness of deep learning techniques in their trading simulation application, highlighting their potential to model noisy financial data and address complex issues in social sciences. In line with the use of deep learning approaches in hybrid models, Ref. [34] introduces a novel time series forecasting model named LS-DL. LS-DL incorporates convolutional neural networks (CNNs), recurrent neural networks (RNNs), and autoregressive (AR) models, leveraging periodic information from time series data for both input and output processing. The deep neural network component focuses on capturing local and long-term dependencies within the time series, whereas the autoregressive model addresses issues related to sensitivity to input scale changes. In experimental evaluations using four public datasets, our model outperformed traditional models and GRU-based models.

Comparing the performance of different deep learning models for financial time series forecasting has attracted significant research attention. For instance, Ref. [12] investigated this by testing and analyzing convolutional networks, evaluating both their conditional and unconditional performance in financial forecasting tasks. The study includes data such as the S&P 500, the volatility index, the CBOE interest rate, and various exchange rates. The comparison extensively evaluates its performance against that of the well-established autoregressive model and a long short-term memory (LSTM) network. The findings demonstrate that the convolutional network excels in regression-type tasks, effectively learning dependencies within and across series without the need for extensive historical data. It proves to be a time-efficient and straightforward alternative to recurrent-type networks, consistently outperforming both linear and recurrent models. Ref. [11] constructs seven different models, with three models utilizing CNN and four models employing LSTM, for both univariate and multivariate time series. The experimental results indicated a significant variation in the accuracy and execution speed of the models. Nevertheless, all the models achieved a high level of accuracy in their forecasting outcomes. Overall, the CNN-based models surpassed the LSTM counterparts in both execution speed and prediction accuracy.

Ref. [35] demonstrated the superiority of deep learning models over traditional machine learning methods in selecting time series forecasting models by combining a CNN with a data augmentation technique and comparing the results with the performance of SVM and traditional time series image algorithms. In contrast, Ref. [24] proposed combining a GRU with a CNN architecture to identify financial market predictions based on return predictive signals. Additionally, the authors trained their model with an attention mechanism (GRU-CNN) and compared its performance with that of traditional deep learning models. The experiments demonstrate that the enhanced GRU-CNN model achieves better predictive performance than previous traditional methods do. Statistically and economically, the existing GRU-based model already delivers good accuracy and higher returns, but the proposed GRU-CNN model performs slightly better than the GRU-based model does.

In merging deep learning models with statistical and econometric time series models, ANNs have been combined with various types of GARCH models [36–39] as well as LSTM [2, 40, 41] for enhancing volatility forecasts. The results of the hybrid models outperformed the standalone deep learning and statistical/econometric models, demonstrating the efficacy of hybrid approaches.

In this work, the DCC model, as an econometric model, and MCNN, as a deep learning model, are employed to construct hybrid models to enhance the forecast of dynamic volatility and correlations among stock markets. The rationale for using CNNs for incorporating econometrics models to forecast time series data is their ability to learn filters that capture recurring patterns within the series, which can then be utilized to predict future values [12]. CNNs can automatically learn and extract features from raw data without requiring prior knowledge or manual feature engineering. Additionally, CNNs can effectively handle noisy time series by filtering out the noise at each successive layer, thereby creating a hierarchy of useful features and retaining only the meaningful features [12].

The contributions of this work are as follows: first, a hybrid model that combines the DCC model with a CNN is proposed. Few studies have hybridized the DCC model with a deep learning model, and none have utilized CNNs for modeling dynamic volatility and correlations between stock market returns. Second, two methods for combining the DCC model with the CNN model are presented. A comparison between the proposed models (DCC-CNN and CNN-DCC) and the single models (DCC and CNN) is conducted. The results show that the best way to combine the DCC model and the CNN model is to use the outputs of the CNN model as inputs to the DCC model. This hybrid DCC-CNN model achieves better performance in forecasting time-varying volatility and correlation among stock markets. Third, the efficacy and efficiency of the CNN model are demonstrated in building hybrid models with volatility forecasting models. The remainder of this paper is organized as follows: Section 2 presents the methodology used in this work, and Section 3 presents the proposed models and empirical results. The discussion and conclusion are explained in Sections 4 and 5, respectively.

2. METHODOLOGY

Dynamic Conditional Correlation (DCC) models have become one of the most widely employed econometric approaches for financial time series volatility modeling. Their widespread adoption stems from demonstrated effectiveness in forecasting both volatilities and correlations across economic and financial variables. The complex, nonlinear correlation structures among financial variables necessitate more flexible models to accurately capture these intricacies. In this context, convolutional neural network (CNN) models provide a substantial advantage because

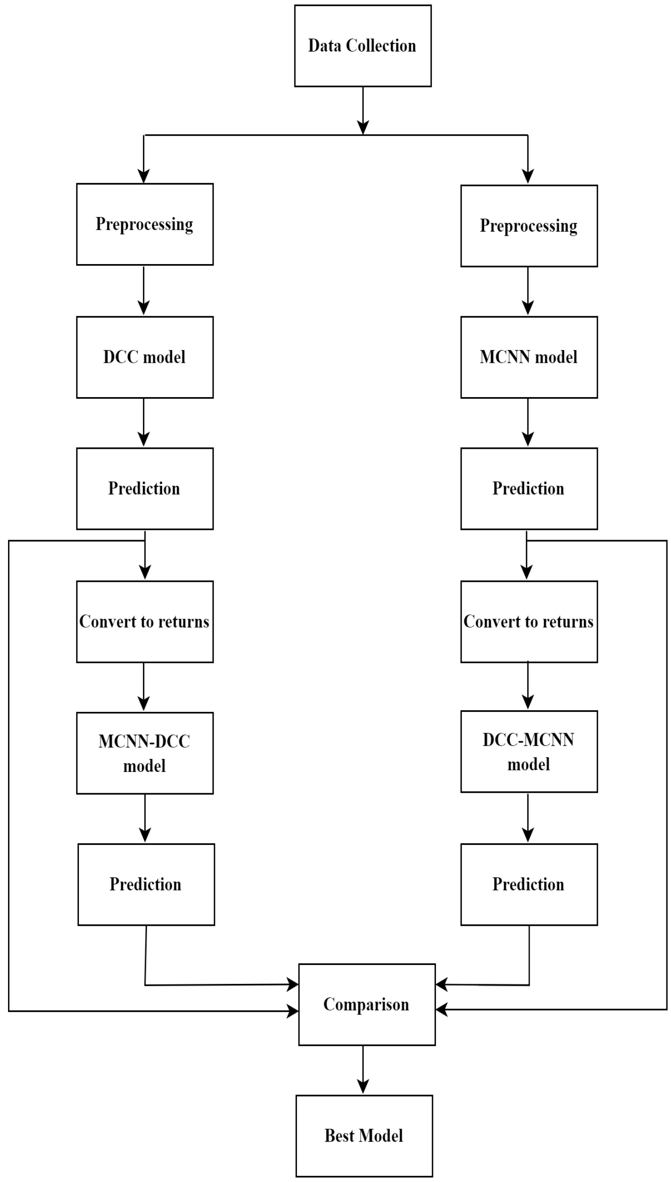

Figure 1. Methodology flowchart.

of their ability to model complex nonlinear relationships without requiring predefined assumptions. To leverage the strengths of both statistical and machine learning models, hybrid models have gained considerable attention from researchers in the field of financial time series modeling.

In this work, we sought to integrate the DCC model with the MCNN model in two distinct ways, as illustrated in Figure 1, following the method of Ref. [42] and compare the performance of these hybrid models against that of the individual DCC and MCNN models via various metrics. The first hybrid model, the DCC-MCNN model, is constructed by converting the outputs from the MCNN model into returns as inputs, see Figure 2. Conversely, the second hybrid model, the MCNN-DCC model, is developed by transforming the outputs of the DCC model into returns and utilizing them as inputs for the MCNN model as it exhibits in Figure 3.

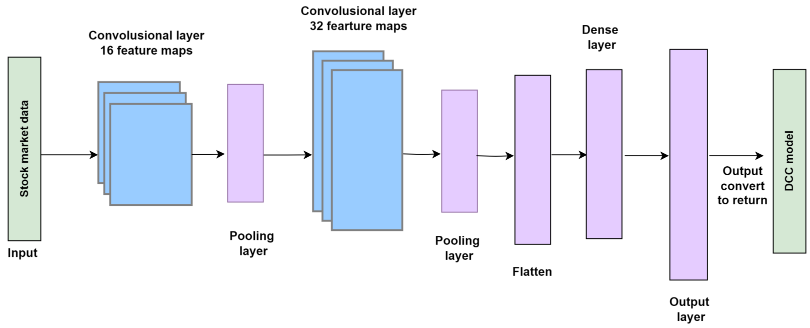

Figure 2. DCC-MCNN model flowchart.

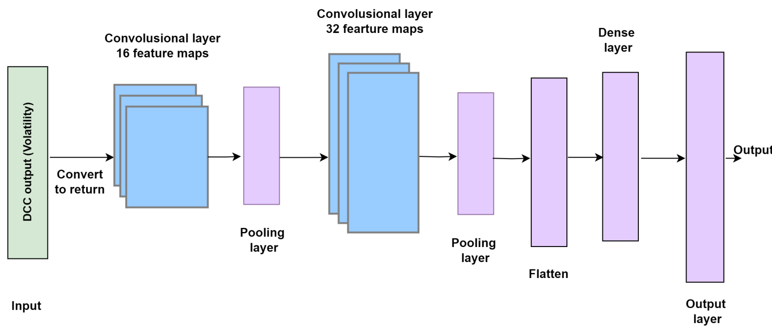

Figure 3. MCNN-DCC model flowchart.

2.1. DCC-GARCH MODEL

The DCC model, proposed by Engle (2002), consists of three different equations structured in the following forms:

$$y_t = \boldsymbol{\mu}_t + a_t\,, \tag{1}$$

$$a_t = \boldsymbol{\Sigma}_t^{\frac{1}{2}} \boldsymbol{\varepsilon}_t\,, \tag{2}$$

$$\boldsymbol{\Sigma}_t = D_t \boldsymbol{\Gamma}_t D_t\,, \tag{3}$$

where in Equation (1), $\boldsymbol{\mu}_t$ is a vector of expected value of y_t, and in Equation (2), $\boldsymbol{\varepsilon}_t$ is a vector of identical independent distribution (*i.i.d*) of errors with $E(\boldsymbol{\varepsilon}_t) = 0$ and $E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t^{'}) = I_k$.

$\boldsymbol{\Gamma}_t = (\text{diag}\{Q_t\})^{-\frac{1}{2}} Q_t (\text{diag}\{Q_t\})^{-\frac{1}{2}}$, in Equation (3), and $Q_t = [q_{ij,t}]$ is the conditional time-dependent variance between series, i and j defines the symmetric DCC model with the following form:

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha e_{t-1} e_{t-1}^{'} + \beta Q_{t-1}\,, \tag{4}$$

$\alpha, \beta > 0$ are non-negative real numbers that satisfy $0 < \alpha + \beta < 1$, and $e_{t-1} e_{t-1}^{'}$ is the lagged function of the standardized residuals. $\bar{Q}$ is unconditional covariance matrix of e_t.

$D_t = \text{diag}\{\sigma_{11,t}^{\frac{1}{2}}, ..., \sigma_{kk,t}^{\frac{1}{2}}\}$ is a diagonal matrix of square roots of the conditional covariance matrix from univariate models.

3. PROPOSED MODEL AND EMPIRICAL RESULTS

3.1. DATA SOURCE AND DESCRIPTION

The dataset used in this work covers daily data from January 1, 2010, to December 30, 2022, comprising 3396 instances, and it was sourced from finance.yahoo.com and includes five stock markets: the JSE. JO from South Africa, S&P 500 from the USA, KLSE from Malaysia, FTSE-100 from the UK, and BSESN from India. Daily returns are computed via the natural logarithm of adjusted closing prices: $y_{it} = \ln\left(\frac{p_{i,t}}{p_{i,t-1}}\right)$ for each market i at time t.

Figures 4 and 5 display the time plots of the adjusted close prices and the log return series for the stock markets. A simultaneous decline approximately 2020 is evident across all markets, attributed to the COVID-19 crisis, as presented in Figure 4. In Figure 5, the series exhibits volatility clusters during certain periods and fluctuates within defined ranges, suggesting statistical stationarity. The volatility clusters are notably pronounced approximately 2020.

Table 1 presents the descriptive statistics and some preliminary tests. The results indicate that the mean values for the stock markets are small. Minimal variation between the stock markets is observed, as evidenced by small standard deviations. For the normal distribution test, the Jarque–Bera test statistics and their p values are less than 0.05, confirming the rejection of normality for the stock markets.

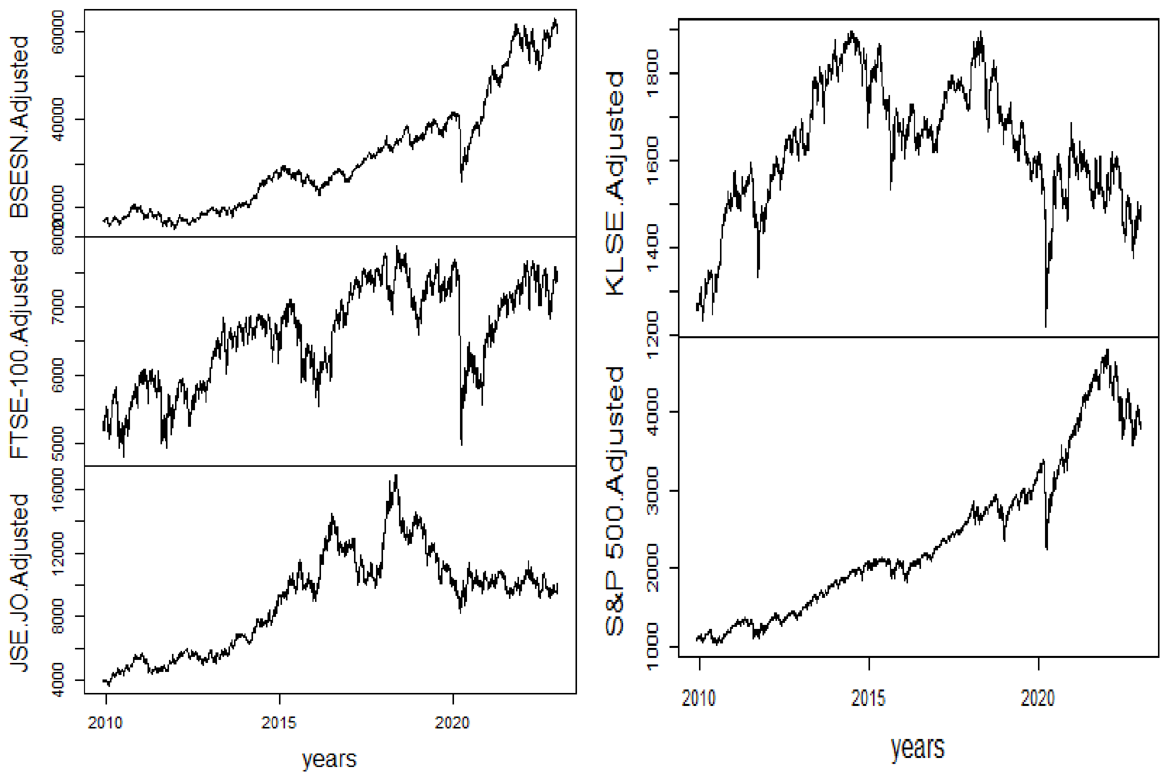

Figure 4. Time plot of daily adjusted close price series.

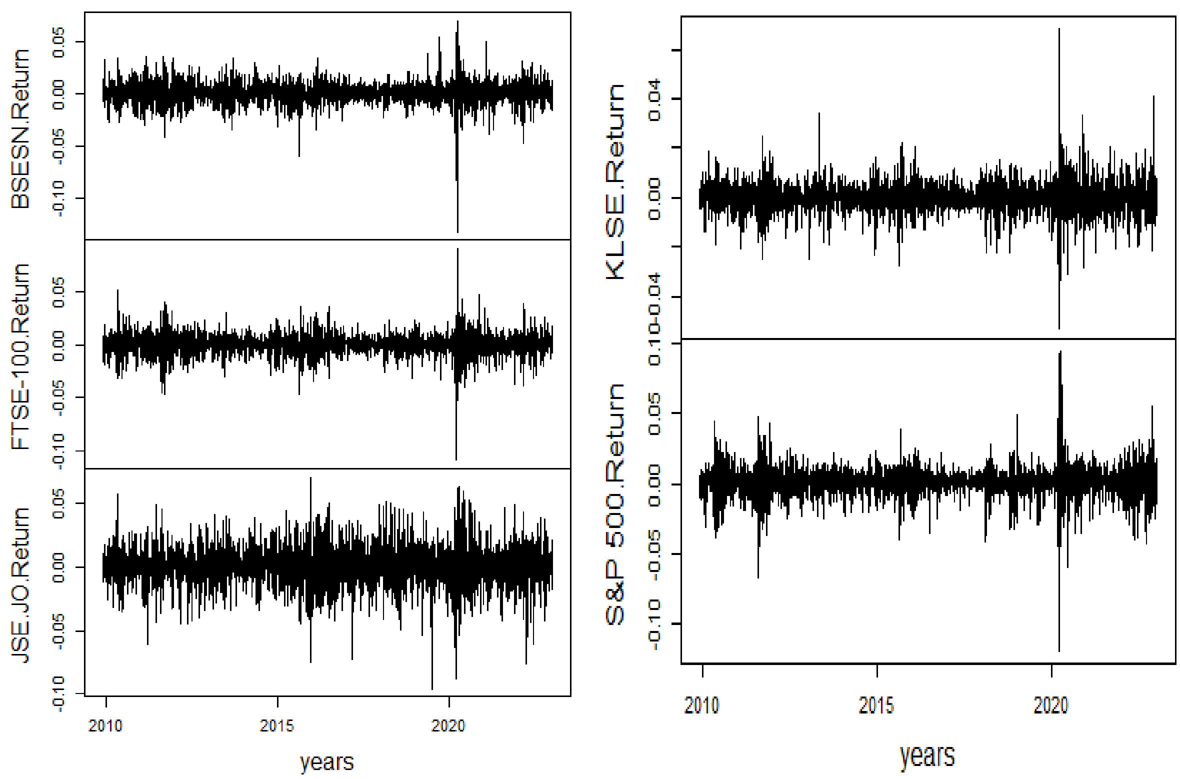

Figure 5. Time plot of the log return series.

3.2. DESIGNING THE MULTIVARIATE CONVOLUTIONAL NEURAL NETWORKS MODEL

Given their capacity to capture intricate nonlinear relationships within data. Artificial neural networks (ANNs) are effective in forecasting. Several factors affect the performance of a neural network model, such as its architecture and parameters, the pre-

Table 1. Summary of descriptive statistics of the return series.

Stock Market	No. of obs	Min	Max	Mean	Std	Skewness	Kurtosis	JB Statistic	p-value
JSE.JO	3396	-0.09569	0.07001	0.000386	0.014853	-0.1455	5.730	1066.595	<0.05
BSESN	3396	-0.13153	0.06980	0.000419	0.010310	-0.8641	15.853	23799.128	<0.05
FTSE-100	3396	-0.10874	0.09053	0.000140	0.010017	-0.4928	12.265	12283.849	<0.05
S&P 500	3396	-0.11984	0.09383	0.000420	0.010943	-0.5043	16.168	24678.412	<0.05
KLSE	3396	-0.05261	0.06851	0.000067	0.006330	-0.0544	12.030	11539.456	<0.05

processing and feature engineering methods used, and the quantity and quality of the input data, among others. Other important factors that improve the predictive performance of ANNs are the training algorithm and the optimization procedure. Various types of artificial neural network (ANN) architectures have been proposed, leading to different kinds of ANNs, such as deep neural networks (DNNs), recurrent neural networks (RNNs), long short-term memory networks (LSTMs), and convolutional neural networks (CNNs).

CNNs consist of two main processing layers that handle the primary computations [11]. The convolutional layers are tasked with identifying key features from the input data, whereas the pooling or subsampling layers summarize these features and extract the most significant ones within a local region. The output from the final pooling layer is then passed to one or more dense layers, where classification or regression tasks are carried out. A multivariate CNN model that uses the previous ten days of data as its input is built. Each of the five variables, stock market returns, is used as a separate channel in a CNN. Two convolutional layers with 16 and 32 filter maps and a kernel size of 3 are deployed in this model. A max pooling layer follows each of the convolutional layers. The output of the max pooling layer is reshaped into a flat one-dimensional vector, and then the flattened vector is allowed to pass into an output layer. Finally, the output layer predicts the stock market return values for the next five days. The model is trained via 100 epochs and a batch size of 32 using a rectified linear unit (ReLU) as the activation function in the convolutional and max pooling layers: and ADAM as the optimizer.

3.3. MODELING HYBRID MODELS OF THE DCC-EGARCH AND MCNN

Volatility in asset markets, such as the stock market, signifies the degree of fluctuation in asset prices, serving as a critical indicator of market uncertainty or risk [2]. Investment firms and private investors leverage volatility [43], to gauge risk through the variability of underlying asset prices. Understanding the volatility and interrelations of stock markets is vital from a practical perspective and remains a focal point of research, notwithstanding the predominant emphasis on modeling stock market volatility. Consequently, extensive studies have been undertaken to model time-varying volatility and correlations among stock markets via statistical and econometric models.

Given the intricate, nonlinear, dynamic, and chaotic nature of stock market data, pinpointing an optimal method for out-of-sample forecasting poses a significant challenge. Researchers often resort to integrating neural networks, which are devoid of prior assumptions, with statistical and econometric models that require assumptions such as homoscedasticity or heteroscedasticity in the variance of the error term. These hybrid models fulfill various objectives, including methodological enhancement, the integration of diverse modeling strategies, and the extraction of multifaceted information.

Table 2. ARCH effect test (volatility test)

Test	Langrange Multiplier	Ljunk-Box	Rank-based	Robust (5%)
Test Statistic	5332.461	11299.34	1891.802	2000.2
p-value	<0.05	<0.05	<0.05	<0.05

Table 3. Information criteria for selecting a DCC model

Criteria	DCC(1,1)	DCC(1,2)	DCC(2,1)	DCC(2,2)
AIC	**-34.35095**	-34.35028	-34.3479	-34.34959
BIC	**-34.26619**	-34.26346	-34.26107	-34.2607

Model integration is crucial for augmenting predictive accuracy and assessing the risk and comovement of stock prices. Consequently, model combination approaches [42] are explored to ascertain the most effective integration methodology.

3.4. DCC-EGARCH MODEL

The initial findings indicate significant volatility in stock market returns and suggest the applicability of the dynamic conditional correlation (DCC) model for data analysis. Consequently, the DCC-EGARCH model is implemented following a conditional heteroscedasticity test, which confirms the presence of the ARCH effect, as detailed in Table 2. Data spanning from January 1, 2010, to January 18, 2021, are utilized for model construction, whereas out-of-sample data from January 19, 2021, to December 30, 2022, are employed for forecasting. The parameters of the model are estimated via the maximum log-likelihood estimation method.

Different orders of DCC-EGARCH models were constructed, and DCC-EGARCH(1,1) emerged as the optimal model on the basis of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), as detailed in Table 3. The parameter estimates, displayed in Table 4, demonstrate the significance of the joint ARCH (α) and GARCH (β) parameters, leading to the rejection of the null hypothesis that these parameters are zero. Additionally, the sum of the joint ARCH (α) and GARCH (β) coefficients is less than one ($0.0061 + 0.9663 = 0.9724 < 1$), validating the DCC model. This result indicates that the correlations between stock market returns are dynamic, refuting the hypothesis of constant conditional correlations.

The highest sum of the ARCH and GARCH coefficients is observed for JSE. JO is 0.973, followed by KLSE- 100 (0.918), BSESN (0.850), FTSE-100 (0.815), and S&P 500 (0.771). These

Table 4. Parameter estimates for the DCC and DCC-MCNN models.

	DCC model						DCC-MCNN model				
Stock	Parameter	Estimate	Std. Error	t value	Pr(>||t||)	ARCH+GARCH	Estimate	Std. Error	t value	Pr(>||t||)	ARCH+GARCH
JSE.JO	mu	0.0004	0.0002	1.751	0.07993	0.973	0.0007	0.0002	4.000	0.00006	0.855
	ar1	0.0216	0.0182	1.190	0.23387		0.0761	0.0211	3.6	0.00032	
	omega	-0.1057	0.0139	-7.634	<0.05		-0.7449	0.1678	-4.441	0.00001	
	alpha1	-0.0140	0.0113	-1.243	0.21394		-0.063	0.0234	-2.694	0.00706	
	beta1	0.9873	0.0016	607.237	<0.05		0.918	0.0184	49.899	<0.05	
	gamma1	0.0990	0.0739	1.339	0.18069		0.3381	0.0471	7.171	<0.05	
	shape	4.3508	0.9211	4.723	<0.05		4.4665	0.3847	11.611	<0.05	
BSESN	mu	0.0004	0.0002	2.501	0.01240	0.854	-0.0153	0.0002	-72.411	<0.05	0.833
	ar1	0.0938	0.0182	5.167	<0.05		0.0794	0.016	4.967	<0.05	
	omega	-0.2482	0.0087	-28.629	<0.05		-0.812	0.191	-4.251	0.00002	
	alpha1	-0.1197	0.0128	-9.336	<0.05		-0.0731	0.0247	-2.961	0.00307	
	beta1	0.9739	0.0009	1075.108	<0.05		0.9063	0.022	41.133	<0.05	
	gamma1	0.1208	0.0192	6.291	<0.05		0.3622	0.0506	7.156	<0.05	
	shape	7.3619	0.9368	7.859	<0.05		4.6111	0.394	11.705	<0.05	
FTSE-100	mu	0.0001	0.0001	0.762	0.44592	0.812	0.0071	0.0002	31.892	<0.05	0.853
	ar1	0.0312	0.0197	1.586	0.11267		0.0794	0.0213	3.731	0.00019	
	omega	-0.2789	0.0042	-66.316	<0.05		-0.7664	0.1939	-3.952	0.00008	
	alpha1	-0.1589	0.0124	-12.853	<0.05		-0.0593	0.0244	-2.431	0.01505	
	beta1	0.9710	0.0001	12559.789	<0.05		0.9123	0.0222	41.177	<0.05	
	gamma1	0.1552	0.0112	13.886	<0.05		0.3587	0.0521	6.884	<0.05	
	shape	7.4480	0.9493	7.846	<0.05		4.5603	0.3807	11.98	<0.05	
S & P 500	mu	0.0006	0.0001	7.252	<0.05	0.771	0.0045	0.0003	17.896	<0.05	0.843
	ar1	-0.0433	0.0158	-2.742	<0.05		0.079	0.0206	3.824	0.00013	
	omega	-0.3372	0.0124	-27.136	<0.05		-0.7537	0.186	-4.052	0.00005	
	alpha1	-0.1945	0.0150	-12.985	<0.05		-0.069	0.0244	-2.824	0.00474	
	beta1	0.9656	0.0012	774.885	<0.05		0.9122	0.0216	42.22	<0.05	
	gamma1	0.2035	0.0228	8.910	<0.05		0.3634	0.0503	7.229	<0.05	
	shape	4.9428	0.4793	10.312	<0.05		4.4859	0.3984	11.261	<0.05	
KLSE-100	mu	0.0001	0.0003	0.481	0.63019	0.919	0.0177	0.0001	133.141	<0.05	0.833
	ar1	0.0874	0.0378	2.312	0.02076		0.0735	0.0208	3.542	0.0004	
	omega	-0.1637	0.0141	-11.611	<0.05		-0.9142	0.199	-4.595	<0.05	
	alpha1	-0.0653	0.0132	-4.938	<0.05		-0.0737	0.0243	-3.035	0.0024	
	beta1	0.9844	0.0014	721.611	<0.05		0.9062	0.0204	44.435	<0.05	
	gamma1	0.1566	0.0202	7.743	<0.05		0.3627	0.0494	7.346	<0.05	
	shape	5.6516	0.5943	9.510	<0.05		4.5869	0.4157	11.034	<0.05	
Joint	[Joint]dcca1	0.0061	0.0026	2.373	0.01764	0.972	0.0433	0.0119	3.636	0.00028	0.975
	[Joint]dccb1	0.9663	0.0216	44.809	<0.05		0.9321	0.0223	41.857	<0.05	
	[Joint]mshape	8.0220	0.4364	18.380	<0.05		4.000	0.1082	36.953	<0.05	

Table 5. Estimated and actual unconditional correlation coefficients.

	DCC model					DCC-MCNN model				
Estimated correlation										
	JSE.JO	BSESN	FTSE-100	S&P 500	KLSE-100	JSE.JO	BSESN	FTSE-100	S&P 500	KLSE-100
JSE.JO	0.948	0.158	0.182	0.124	0.158	0.945	0.932	0.931	0.929	0.933
BSESN		0.997	0.370	0.262	0.297		0.957	0.943	0.940	0.946
FTSE-100			0.995	0.558	0.236			0.954	0.938	0.942
S&P 500				0.995	0.137				0.950	0.943
KLSE-100					1					0.959
Actual correlation (Pearson)										
	JSE.JO	BSESN	FTSE-100	S&P 500	KLSE-100	JSE.JO	BSESN	FTSE-100	S&P 500	KLSE-100
JSE.JO	1	0.182	0.225	0.147	0.162	1	0.984	0.984	0.984	0.984
BSESN		1	0.456	0.305	0.376		1	0.989	0.989	0.991
FTSE-100			1	0.584	0.281			1	0.989	0.989
S&P 500				1	0.132				1	0.991
KLSE-100					1					1

Table 6. Information criteria for selecting a DCC-MCNN model.

Criteria	DCC-MCNN(1,1)	DCC-MCNN(1,2)	DCC-MCNN(2,1)	DCC-MCNN(2,2)
AIC	-47.92953	**-47.93127**	-47.92868	-47.93042
BIC	**-47.82959**	-47.8289	-47.82631	-47.82561

sums reflect the persistence in conditional variances with JSE. JO has the greatest persistence, and the S&P 500 has the least persistence. Notably, all the ARCH coefficients, which measure the own-volatility spillover effect, are negative: -0.1951 for S&P 500, -0.1563 for FTSE-100, -0.1235 for BSESN, -0.0661 for KLSE-100, and -0.0140 for JSE.JO. The GARCH coefficients, indicating persistence, are positive and ranked as follows: JSE. JO (0.987), KLSE-100 (0.984), BSESN (0.974), FTSE-100 (0.972), and S&P 500 (0.966).

Table 5 compares the estimated and actual unconditional correlation coefficients. Within the DCC model, the highest estimated correlation is between FTSE-100 and S&P 500 (0.559), closely matching the actual (Pearson) correlation of 0.584. The lowest estimated correlation is 0.124 between JSE.JO and S&P 500, while the lowest actual correlation is 0.132 between S&P 500 and KLSE-100. For the JSE.JO stock market, the highest correlation is with FTSE-100, and the lowest is with S&P 500, consistent in both estimated and actual correlations.

The DCC-MCNN model demonstrates high unconditional correlations among stock market pairs in both estimated and actual (Pearson) correlations, attributed to CNN's efficacy in feature extraction across variables.

3.5. HYBRID MODEL

Table 6 presents the information criteria for various orders of the hybrid DCC-MCNN models that were constructed. Based on the Bayesian Information Criterion (BIC), the DCC-MCNN model of order (1,1) was chosen as the most suitable model due to its simplicity.

The hybrid DCC-MCNN(1,1) model integrates CNN outputs as inputs for the DCC model, enhancing its ability to capture complex dependencies and correlations in the data. Parameters are estimated via the maximum log-likelihood estimation method, with estimates detailed in Table 4. Like those of the DCC-EGARCH model, the ARCH (α) and GARCH (β) coefficients are significant, and their sum is less than one ($0.0433 + 0.9321 = 0.9754 < 1$). The persistence measurements for conditional variances are 0.855 for JSE.JO, 0.853 for FTSE-100, 0.843 for S&P 500, 0.833 for BSESN, and 0.833 for KLSE-100. The spillover effect coefficients are negative and significant: -0.063 for JSE.JO, -0.0731 for BSESN, -0.0593 for FTSE-100, -0.069 for S&P 500, and -0.0737 for KLSE-100. The persistence coefficients are 0.918 for JSE.JO, 0.906 for BSESN, 0.912 for FTSE-100, 0.912 for S&P 500, and 0.906 for KLSE-100.

The performance of the models under consideration is compared based on metrics such as RMSE, MAE, and RMAE. The metrics are employed to evaluate and contrast the performance of the models within both in-sample and out-of-sample contexts. The discrepancy where in-sample errors are lower than out-of-sample (forecast) errors indicates the effective performance of the models trained on stock market return data. Table 7 outlines the in-sample and out-of-sample accuracies of the four models. Compared with the single models, the hybrid models generally exhibited the lowest values for both in-sample and out-of-sample accuracy metrics, such as the RMSE and MAE, with the hybrid DCC-MCNN model showing superior performance over the hybrid MCNN-DCC model. With respect to the RMAE, the hybrid MCNN-DCC model demonstrated the lowest values for both in-sample and out-of-sample accuracy, followed by the single MCNN model, the hybrid DCC-MCNN model, and finally the DCC models.

4. DISCUSSION

Analyzing model performance via in-sample and out-of-sample metrics provides valuable insights into their effectiveness in predicting stock market returns. The lower in-sample errors compared with out-of-sample forecasts suggest that while the models fit historical data well, their ability to generalize to new data varies.

The hybrid model, which integrates convolutional neural network outputs into a dynamic conditional correlation model, consistently demonstrates stronger performance across both in-sample and out-of-sample accuracy measures such as RMSE and MAE. Specifically, the hybrid DCC-MCNN model emerges as the top performer among the evaluated models, surpassing

Table 7. In-sample and out-of-sample accuracy of the analysis.

Country	DCC model		MCNN model		MCNN-DCC model		DCC-MCNN model	
	In-sample	Out-of-sample	In-sample	Out-of-sample	In-sample	Out-of-sample	In-sample	Out-of-sample
Root mean square error (RMSE)								
JSE.JO	0.22285	0.22574	0.01199	0.02179	0.00708	0.01714	0.00402	0.00755
BSESN	0.15509	0.15010	0.00944	0.02230	0.00581	0.01516	0.00518	0.00518
FTSE-100	0.15304	0.13582	0.01005	0.02367	0.01153	0.02682	0.00494	0.00500
S&P 500	0.16246	0.18400	0.00805	0.01942	0.00493	0.01558	0.00542	0.00467
KLSE-100	0.09284	0.10124	0.00726	0.01978	0.00404	0.02027	0.00300	0.00669
Mean absolute error (MAE)								
JSE.JO	0.21059	0.21732	0.00751	0.01289	0.00610	0.01269	0.00315	0.00647
BSESN	0.13632	0.14060	0.00606	0.01384	0.00435	0.01161	0.00404	0.00415
FTSE-100	0.13414	0.12651	0.00638	0.01463	0.00980	0.02112	0.00388	0.00396
S&P 500	0.13150	0.16979	0.00474	0.01210	0.00371	0.01246	0.00424	0.00341
KLSE-100	0.08161	0.09786	0.00453	0.01145	0.00314	0.01430	0.00236	0.00554
Relative mean absolute error (RMAE)								
JSE.JO	93.2952	95.7719	1.2953	2.2180	0.9540	2.0028	31.0641	65.1487
BSESN	93.6042	93.6124	1.0460	2.3813	0.6801	1.8313	32.0115	34.8822
FTSE-100	93.6197	92.9506	1.1006	2.5178	1.5326	3.3331	31.8479	33.6640
S&P 500	93.5063	94.6517	0.8171	2.0822	0.5805	1.9659	32.2774	27.2341
KLSE-100	93.4685	91.0712	0.7823	1.9699	0.4910	2.2556	32.1146	79.0470

both the single model approaches and the hybrid MCNN-DCC model. This superiority highlights the benefits of leveraging the advanced feature extraction capabilities of CNNs to enhance the predictive accuracy of financial models. This result is in line with Refs. [2, 32, 41, 44, 45] in terms of the hybrid models surpassing the single models and hence enhancing the forecasting of statistical models. Furthermore, the hybrid DCC-CNN model outperforms the hybrid CNN-DCC model in integrating with the DCC model. This is because the DCC-CNN model preserves the DCC model's characteristics of dynamic volatility and correlation, while the CNN-DCC model does not. This finding is consistent with Ref. [42].

From an economic perspective, the superior performance of the DCC-CNN hybrid model can be attributed to its ability to capture complex nonlinear relationships and temporal dependencies inherent in financial markets. Volatility and return correlations across international markets are often driven by macroeconomic news, investor sentiment, and inter-market spillover effects, which may exhibit non-obvious patterns. By preserving the DCC model's structure, the hybrid framework retains crucial information about time-varying volatilities and cross-asset co-movements. Meanwhile, the CNN component contributes by detecting localized patterns or abrupt structural changes in the data, features that traditional econometric models may overlook. This synergy enables more economically meaningful forecasts, particularly under volatile or regime-switching conditions where adaptability and precision are critical.

These results emphasize the effectiveness of incorporating convolutional neural network outputs in enhancing the predictive capabilities of dynamic conditional correlation models for financial forecasting.

5. CONCLUSION

Overall, the models evaluated demonstrate varying levels of performance when applied to stock market return data. The comparison of in-sample and out-of-sample metrics reveals that while in-sample errors tend to be lower, indicating a good fit, forecasting accuracy is generally lower. The hybrid models consistently exhibit superior performance in terms of both in-sample and out-of-sample accuracy metrics such as RMSE and MAE, with the hybrid DCC-MCNN model showing particular strength compared to the hybrid MCNN-DCC and single model approaches.

For investors and financial analysts, these results highlight the practical value of adopting hybrid models like DCC-MCNN to enhance portfolio allocation, risk assessment, and trading strategies. The model's ability to capture nonlinear dependencies and dynamic correlations can lead to more robust volatility forecasts, improving hedging effectiveness and asset pricing accuracy. Analysts should prioritize integrating machine learning techniques with traditional econometric models, as this synergy offers a competitive edge in anticipating market shifts while mitigating overfitting risks inherent in purely data-driven approaches.

The findings underscore the importance of considering model complexity and the integration of innovative techniques like CNNs in financial forecasting. By effectively capturing complex relationships and patterns in stock market data, hybrid models like DCC-MCNN can potentially improve decision-making processes in investment strategies and risk management. Future research could focus on refining these hybrid models further and exploring their applicability across different financial markets and economic conditions, such as crises, by taking into account the pre-crises, crises, and post-crises periods.

DATA AVAILABILITY

The data are available with the corresponding author upon request.

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