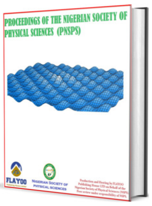
Proceedings of the Nigerian Society of Physical Sciences 3 (2026) 315

Published by Nigerian Society of Physical Sciences. Hosted by FLAYOO Publishing House LTD

Proceedings of the Nigerian Society of Physical Sciences

Journal Homepage: https://flayoophl.com/journals/index.php/pnspsc



Pursuit differential game with high-order dynamic

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ABSTRACT

This study examines a pursuit–evasion differential game with finitely many pursuers and a single evader moving inside a nonempty compact convex region of $\mathbb{R}^2$ The dynamics of all participants are governed by ordinary differential equations of degrees n and m, while their control inputs satisfy coordinate-wise integral constraints. Capture is said to occur when at least one pursuer reaches the same position as the evader. Sufficient conditions ensuring that the pursuers can guarantee capture are derived, and constructive strategies are proposed to realize this outcome.

Keywords: Pursuer, Evader, Coordinate-wise integral constraint, Convex set.

DOI:10.61298/pnspsc.2026.3.315

1. INTRODUCTION

The theory of differential game provides a comprehensive framework for simulating conflict-controlled dynamical systems governed by differential equations. The Pursuit-evasion problem aims to establish conditions for capture or escape, determine pursuit time, and formulate permissible strategies that the players can use. Such problems [1–13] have drawn the attention in the literature due to their theoretical and practical significance. Ibragimove [1] studied a pursuit-evasion game with integral constraints on the control input of the players, in which a single evader and single pursuer evolved in a closed convex region $N \subset \mathbb{R}^n$, the pursuit time was derived, the condition for both pursuit and evasion ware established, and players strategies were obtained. This study was later extended in Ref. [2] to the game with two pursuers and single evader in the plane, under which control resources of the players obeyed integral constraint on each coordinate, where adequate conditions for the pursuit's completion and corresponding pursuers strategy were established.

Pursuit games with multiple pursuers have also been widely studied. In Ref. [3], the first-order dynamic pursuit-evasion game of three pursuers and a single evader was analyzed under geometric control constraints in $\mathbb{R}^2$, and effective pursuer strategies were proposed. The case of many pursuers and single evader in $\mathbb{R}^2$ subject to integral constraints imposed on each coordinate-wise of the control input of the players was considered in [4, 5], where sufficient conditions for capture were obtained and constructive pursuer strategies were developed. An evasion-oriented formulation under similar constraints was examined in [6], with explicit evader strategies provided.

Further extensions include the work in Ref. [7], where pursuit

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was studied within a closed convex region in a $\mathbb{R}^2$ using properties of convex sets, and [8], which addressed a pursuit game on a cylindrical surface and reduced it to an equivalent planar formulation with geometric constraints.

Research Gap: The existing literature predominantly addresses pursuit–evasion games with first-order player dynamics, particularly in $\mathbb{R}^2$, under integral or coordinate-wise integral constraints.

Contribution of the Present Work. In contrast, the present study considers a pursuit differential game involving many pursuers and single evader in $\mathbb{R}^2$, where pursuers are governed by n^{th}-order dynamics and the evader by m^{th}-order dynamics, with $n < m$. Sufficient conditions for the completion of pursuit were derived, and explicit pursuer strategies were constructed, thereby extending existing first-order results to higher-order dynamical systems.

2. DESCRIPTION OF THE RESEARCH PROBLEM

We will consider the differential game which includes many Pursuers and Evader whose state vectors are x and y, and whose velocity vector are u and v, respectively in the space $\mathbb{R}^2$. Let movements of the pursuers and the evader be expressed by the differential equations and initial conditions

$$\begin{aligned} &P_i : \dot{x}_i = \frac{(\theta - t)^{n-1}}{(n-1)!} u_i(t), \quad x_i(0) = x_{i0}, \quad I = \{1, 2, \cdots\}, \\ &E : \dot{y} = \frac{(\theta - t)^{m-1}}{(m-1)!} v(t), \quad y(0) = y_0, \end{aligned} \tag{1}$$

correspondingly, where $x_{i0} = x_i^0 + \theta x_i^1 + \frac{\theta^2}{2!} x_i^2 + \cdots + \frac{\theta^{n-1}}{(n-1)!} x_i^{n-1}$, $y_0 = y^0 + \theta y^1 + \frac{\theta^2}{2!} y^2 + \cdots + \frac{\theta^{m-1}}{(m-1)!} y^{m-1}$, $n, m \in \mathbb{N}$ with $1 \leq n < m$, $x_i^{(k)}, u_i, \in \mathbb{R}^2$, $k = 0, 1, \ldots, n-1$, and $y^{(k)}, v \in \mathbb{R}^2$, $k = 0, 1, \ldots, m-1$. the players parameters control are the velocity $u_i = (u_{i_1}, u_{i_2})$ and $v = (v_1, v_2)$ respectively and they depend on time $t \geq 0$. From (1) we obtained the following state equations of the players.

$$\begin{aligned} x_i(\theta) &= x_{i0} + \int_0^{\theta} \frac{(\theta - t)^{n-1}}{(n-1)!} u_i(t) dt, \\ y(\theta) &= y_0 + \int_0^{\theta} \frac{(\theta - t)^{m-1}}{(m-1)!} v(t) dt. \end{aligned}$$

Definition 2.1. Admissible control of the i^{th} pursuer is a measurable function $u_i(t) = (u_{i1}(t), u_{i2}(t))$; $t \geq 0$, defined as:

$$\int_0^{\infty} \left| u_{ij}(t) \right|^2 dt \leq \rho_{ij}^2, \quad i = 1, \cdots N, \quad j = 1, 2, \tag{2}$$

where ρ_{ij}, are assigned positive value.

Definition 2.2. Admissible control of the evader is a measurable function $v(t) = (v_1(t), v_2(t))$; $t \geq 0$ defined as

$$\int_0^{\infty} \left| v_j(t) \right|^2 dt \leq \sigma_j^2, \quad j = 1, 2, \tag{3}$$

where σ_j, are assigned positive number.

Definition 2.3. The strategy of the i^{th} pursuer, is a Borel measurable function $U_i(x_i, y, v) = (U_{i1}(x_i, y, v), U_{i2}(x_i, y, v)$, $U_i : \mathbb{R}^6 \to \mathbb{R}^2$,such that for any control of the evader $v(t), t \geq 0$, the initial value problem

$$\begin{aligned} &\frac{d^n x_i}{dt^n} = U_i(t, x_i, y, v), \quad x_i(0) = x_i^0 \ , \\ &\frac{dx_i}{dt}(0) = x_i^1, \ldots, \frac{d^{n-1} x_i}{dt^{n-1}}(0) = x_i^{n-1}, \quad i \in I, \\ &\frac{d^m y}{dt^m} = v(t), \qquad y(0) = y^0, \\ &\frac{dy}{dt}(0) = y^1, \ldots, \frac{dy^{m-1}}{dt^{m-1}}(0) = y^{m-1}, \end{aligned} \tag{4}$$

has exactly one solution $(x_i(t), y(t))$ and the following coordinate-wise integral constraint:

$$\int_0^{\infty} \left| U_{ij}(x_i(s), y(s), v(s)) \right|^2 ds \leq \rho_{i1}^2, \quad i = 1, \ldots m, \quad j = 1, 2, \tag{5}$$

hold.

Definition 2.4. In the game (1), pursuit is said to be accomplished from the initial position $\{x_{10}, \ldots, x_{m0}, y_0\}$ within the time interval $[0, T]$, under conditions in Definitions (2.1) and (2.2), if there exist admissible strategies U_i, $i = 1, \ldots, N$, of the pursuers such that for any control $v = v(.)$ of the evader, there exist an index $i \in \{1, \ldots, N\}$ and $t \in [0, T]$, for which the equality $x_i(t) = y(t)$ holds.

3. RESEARCH OUTCOME

Theorem 3.1. *For pursuit to be guaranteed in the game* (2.1) *within a finite time T, starting from any initial position, it is sufficient that*

$$\sum_{i=1}^{m} \rho_{ij}^2 > \sigma_j^2, \quad j \in \{1, 2\}, \tag{6}$$

subject to the conditions in Definitions (2.1) *and* (2.2).

Proof. Assume that the inequality holds for a fixed index $j = j^*$, that is,

$$\sum_{i=1}^{m} \rho_{ij^*}^2 > \sigma_{j^*}^2.$$

We now define auxiliary quantities σ_{ij^*} for each pursuer i as

$$\sigma_{ij^*} = \frac{\sigma_{j^*}}{\rho_{j^*}} \rho_{ij^*}, \quad i = 1, 2, \ldots, N,$$

where

$$\rho_{j^*} = \left(\sum_{i=1}^{m} \rho_{ij^*}^2 \right)^{1/2}.$$

Justification: This formula is introduced by construction. It defines σ_{ij^*} proportional to the corresponding ρ_{ij^*}, with the proportionality constant chosen so that the combined magnitude of σ_{ij^*} matches the evader's bound:

$$\left(\sum_{i=1}^{m} \sigma_{ij^*}^2 \right)^{1/2} = \sigma_{j^*}.$$

By substituting $\sigma_{ij^*} = r\rho_{ij^*}$, the proportionality constant r is determined as $r = \sigma_{j^*}/\rho_{j^*}$, which directly leads to the formula above. This ensures that the allocated "share" for each pursuer is consistent and respects the total bound of the evader.

From the inequality $\sum_{i=1}^{m}\rho_{ij^*}^2 > \sigma_{j^*}^2$, it follows that

$$\rho_{j^*} > \sigma_{j^*} \quad \text{and hence} \quad \frac{\sigma_{j^*}}{\rho_{j^*}} < 1.$$

Substituting this into the definition of σ_{ij^*} gives

$$\sigma_{ij^*} < \rho_{ij^*}, \quad i = 1, 2, \dots, m.$$

□

Let x_0 and y_0 be points in the set N such that

$$|x_0 - y_0| = \operatorname{diam} N := \max_{x,y\in N}|x - y|.$$

Choosing x-axis to pass through the points x_0 and y_0. Denote

$$d = |x_0 - y_0|, \qquad c = \max_{(\zeta,\eta)\in N}|\eta|.$$

We assume without loss of generality that, all pursuers initially lie on the x_1-axis. If not, they can be easily transferred to that position by using the following control inputs.

Let $\rho = \rho_{sj}$ for some $j \in \{1, 2\}$. Suppose that, the pursuer's P_s initial position takes the form $x_s^0 = (x_{s1}^0, 0)$. Otherwise, they can be transformed to that position by applying the control below, i.e. $x_s(\tau_0) = (x_{s1}^0, 0)$, for the time interval $[0, \tau_0]$.

$$U_{sj}(t) = \begin{cases} 0, & \mathit{for}\ j = 1, \\ \frac{-x_{s2}^0(n-1)}{\tau_0(\tau_0-t)^{n-1}}, & \mathit{for}\ j = 2, \end{cases} \tag{7}$$

where $\tau_0 = \max_{s\in S}\dfrac{|x_{s2}^0|^2(n-1)^2}{\rho_{s2}^2(3-2n)^2}$. Indeed for $j = 2$ and $U_{sj}(t) \neq 0$, we have

$$\begin{aligned} x_{s2}(\tau_0) &= x_{s2}^0 + \int_0^{\tau_0}\frac{(\tau_0 - t)^{n-1}}{(n-1)!}u_{s2}(t)dt \\ &= x_{s2}^0 + \int_0^{\tau_0}\frac{(\tau_0 - t)^{n-1}}{(n-1)!}\left(\frac{-x_{s2}^0(n-1)}{\tau_0(\tau_0 - t)^{n-1}}\right)dt \\ &= x_{s2}^0 + \int_0^{\tau_0}\left(-\frac{x_{s2}^0}{\tau_0}\right) \\ &= x_{s2}^0 - x_{s2}^0 = 0. \end{aligned}$$

The control equation (7) is admissible, indeed

$$\begin{aligned} \int_0^{\tau_0}|U_{s2}(t)|^2\,dt &= \int_0^{\tau_0}\left|\frac{-x_{i2}^0(n-1)!}{\tau_0(\tau_0 - t)^{n-1}}\right|^2 dt \\ &= \frac{|x_{i2}^0|^2(n-1)!^2\tau_0^{3-2n}}{(3-2n)\tau_0^2} = \frac{|x_{i2}^0|^2(n-1)!^2}{(3-2n)\tau_0^{2n-1}} \\ &\le \frac{|x_{i2}^0|^2(n-1)!^2}{(3-2n)\tau_0} \le \frac{|x_{i2}^0|^2(n-1)!^2}{(3-2n)\left(\frac{|x_{s2}^0|^2(n-1)!^2}{\rho_{s2}^2(3-2n)^2}\right)} \\ &= \rho_{s2}^2(3-2n) \le \rho_{s2}^2. \end{aligned}$$

The following stage of the game, we will consider $\rho_{s2}^* := \frac{\sqrt{3}}{2}\rho_{s2}$, instead of ρ_{s2} . In this stage, for the time $[\tau_0, \tau_1]$, the s^{th} pursuer is to apply the following control on the first coordinate:

$$U_{sj}(t) = \begin{cases} 0, & j = 2, \\ \dfrac{sgn(y_1(\tau_0) - x_{s1}(\tau_0))(n-1)!d}{(\tau_1 - \tau_0)(\tau_1 - t)^{n-1}}, & j = 1, \end{cases} \tag{8}$$

where $\tau_1 = \tau_0 + \dfrac{(n-1)^2d^2}{(1-2n)^2(\rho_{s1} - \sigma_{s1})^2}$. indeed, this control will assure that $x_{s1}(\tau_1) = y_1(\tau_1)$. Certainly,

$$\begin{aligned} x_{s1}(\tau_1) &= x_{s1}(\tau_0) + \int_{\tau_0}^{\tau_1}\frac{(\tau_0 - t)^{n-1}}{(n-1)!}U_{s1}dt \\ &= x_{s1}(\tau_0) \\ &+ \int_{\tau_0}^{\tau_1}\frac{(\tau_0 - t)^{n-1}}{(n-1)!}\left(\frac{sgn(y_1(\tau_0) - x_{s1}(\tau_0))(n-1)!d}{(\tau_1 - \tau_0)(\tau_1 - t)^{n-1}}\right)dt \\ &= x_{s1}(\tau_0) + \frac{d}{\tau_1 - \tau_0}\int_{\tau_0}^{\tau_1}dt \\ &= x_{s1}(\tau_0) + \frac{d}{(\tau_1 - \tau_0)}(\tau_1 - \tau_0) \\ &= x_{s1}(\tau_0) + d \\ &= y_1(\tau_1). \end{aligned}$$

The following stage of the game on the time interval $[\tau_1, \tau_2]$ pursuer uses the following control fir the first to second coordinates

$$U_{i^*j}(t) = (v_1(t), U_{s2}(t)),$$

where $U_{s2}(t) = \dfrac{sgn(y_1(\tau_0) - x_{s1}(\tau_0))(n-1)!c}{(\tau_1 - \tau_0)(\tau_1 - t)^{n-1}}$ and $\tau_2 = \tau_0 + \dfrac{(n-1)^2c^2}{(1-2n)^2(\rho_{s1} - \sigma_{s1})^2}$.

This strategy will ensure that $x_{s1}(\tau_2) = y_1(\tau_2)$ and $x_{s2}(\tau_2) = y_2(\tau_2)$.

$$\begin{aligned} x_{s1}(\tau_1) &= x_{s1}(\tau_0) + \int_{\tau_0}^{\tau_1}\frac{(\tau_0 - t)^{n-1}}{(n-1)!}U_{s1}dt \\ &= x_{s1}(\tau_0) \\ &+ \int_{\tau_0}^{\tau_1}\frac{(\tau_0 - t)^{n-1}}{(n-1)!}\left(\frac{sgn(y_1(\tau_0) - x_{s1}(\tau_0))(n-1)!c}{(\tau_1 - \tau_0)(\tau_1 - t)^{n-1}}\right)dt \\ &= x_{s1}(\tau_0) + \frac{c}{\tau_1 - \tau_0}\int_{\tau_0}^{\tau_1}dt \\ &= x_{s1}(\tau_0) + \frac{c}{(\tau_1 - \tau_0)}(\tau_1 - \tau_0) \\ &= x_{s1}(\tau_0) + c \\ &= y_1(\tau_1). \end{aligned}$$

The above strategy is admissible, this can be shown as follow:

For the first coordinate, we have

$$\begin{aligned}
\int_0^{\tau_0} |U_{s1}(t)|^2\,dt &= \int_0^{\tau_0} \left|\frac{sgn(y_1(\tau_0)-x_{s1}(\tau_0))(n-1)!d}{(\tau_1-\tau_0)(\tau_1-t)^{n-1}}\right|^2 dt \\
&+ \sum_{r=1}^{n-1}\int_{\tau_r}^{\tau_{r+1}} |v_1(t)|^2 dt \\
&= \frac{(n-1)!d^2}{(\tau_1-\tau_0)^2}\int_{\tau_0}^{\tau_1}\frac{1}{(\tau_1-t)^{2(n-1)}}dt \\
&+ \sum_{r=1}^{n-1}\int_{\tau_r}^{\tau_{r+1}} |v_1(t)|^2 dt \\
&\le \frac{(n-1)!d^2}{(\tau_1-\tau_0)^2}\int_{\tau_0}^{\tau_1}(\tau_1-t)^{-2(n-1)}dt + \sigma_1^2 \\
&= \frac{(n-1)!d^2}{(1-2n)(\tau_1-\tau_0)^{2n-1}} + \sigma_1^2 \\
&\le \frac{(n-1)!d^2}{(1-2n)(\tau_1-\tau_0)} + \sigma_1^2 \\
&\le \frac{(n-1)!d^2}{(1-2n)\left(\tau_0 + \frac{(n-1)^2d^2}{(1-2n)^2(\rho_{s1}-\sigma_{s1})^2} - \tau_0\right)} + \sigma_1^2 \\
&= (\rho_{s1}-\sigma_1)^2(1-2n) + \sigma_1^2 \\
&\le (\rho_{s1}^2-\sigma_1^2)(1-2n) + \sigma_1^2 \\
&\le \rho_{s1}^2 - \sigma_1^2 + \sigma_1^2 = \rho_{s1}^2.
\end{aligned}$$

For the second coordinate, we have:

$$\begin{aligned}
\int_{\tau_0}^{\tau_2} |U_{s2}(t)|^2\,dt &= \int_{\tau_1}^{\tau_2} \left|\frac{sgn(y_2(\tau_1)-x_{s2}(\tau_1))(n-1)!c}{(\tau_2-\tau_1)(\tau_2-t)^{n-1}}\right|^2 dt \\
&+ \sum_{r=2}^{n-1}\int_{\tau_r}^{\tau_{r+1}} |v_2(t)|^2 dt \\
&= \frac{(n-1)!c^2}{(\tau_2-\tau_1)^2}\int_{\tau_1}^{\tau_2}\frac{1}{(\tau_2-t)^{2(n-1)}}dt \\
&+ \sum_{r=2}^{n-1}\int_{\tau_r}^{\tau_{r+1}} |v_2(t)|^2 dt \\
&\le \frac{(n-1)!c^2}{(\tau_2-\tau_1)^2}\int_{\tau_1}^{\tau_2}(\tau_2-t)^{-2(n-1)}dt + \sigma_2^2 \\
&= \frac{(n-1)!c^2}{(1-2n)(\tau_2-\tau_1)^{2n-1}} + \sigma_2^2 \\
&\le \frac{(n-1)!c^2}{(1-2n)(\tau_2-\tau_1)} + \sigma_2^2 \\
&\le \frac{(n-1)!c^2}{(1-2n)\left(\tau_1 + \frac{(n-1)^2d^2}{(1-2n)^2(\rho_{s2}-\sigma_{s2})^2} - \tau_0\right)} + \sigma_2^2 \\
&= (\rho_{s2}-\sigma_2)^2(1-2n) + \sigma_2^2 \\
&\le (\rho_{s2}^2-\sigma_2^2)(1-2n) + \sigma_2^2 \\
&\le \rho_{s2}^2 - \sigma_2^2 + \sigma_2^2 = \rho_{s2}^2.
\end{aligned}$$

The existence of the times τ_i, $i \in \{1,2\}$, is guaranteed. In particular, the existence of τ_2 follows from the control applied by pursuer to second coordinate over the interval $[\tau_1, \tau_2]$. During this interval, pursuer drives its second coordinate toward corresponding coordinate of evader. In response, evader may attempt to increase the separation by moving in the opposite direction. However, since the evader's motion is confined to the compact set C, once its second coordinate reaches the boundary of C, it must either remain there or reverse its direction of motion.

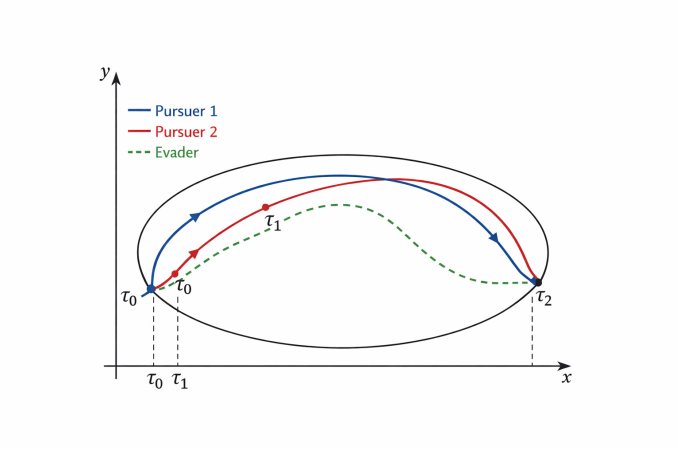

Figure 1. Trajectories of two pursuers (solid lines) and a single evader (dashed line) in a closed convex set. Phase times τ_0, τ_1, τ_2 are marked to illustrate the sequential alignment and capture process.

In either case, the relative motion between the pursuer and the evader ensures that the equality

$$x_{s2}(\tau_2) = y_2(\tau_2)$$

is attained within a finite time τ_2. Hence, all times τ_i, $i \in \{1,2\}$, exist, which conclude the theorem's proof.

4. ILLUSTRATIVE EXAMPLE AND TRAJECTORY ANALYSIS

To visualize the effectiveness of the proposed pursuit strategy under coordinate-wise integral constraints, we consider a numerical example with two pursuers and one evader evolving within the compact convex set

$$N = \left\{(x,y) \in \mathbb{R}^2 \mid \frac{x^2}{9} + \frac{y^2}{4} \le 1\right\}.$$

The control limits and starting positions are selected as:

$$m = 2, \quad \rho_{11} = 1, \quad \rho_{21} = 1.1, \quad \sigma_1 = \sqrt{2}, \quad \rho_{12}, \rho_{22} > 0.$$

Completion of pursuit can be achieved from theorem 1, under these parameters

4.1. INTERPRETATION OF THE TRAJECTORY PLOT

The plot in Figure 1 illustrates how the pursuer's strategy is implemented in stages:

- Stage τ_0: Using only their initial coordinates, the pursuers align their starting positions along the x-axis.
- Stage τ_1: The first coordinate of the pursuers coincides with that of the evader in the same coordinate, that is, $x_1 = y_1$
- Stage τ_2: Pursuit is completed, under the coordinate-wise integral constraints, the coordinates of at least one pursuer coincide with that of the evader, that is, $x_1 = y_1$ and $x_2 = y_2$.

It is clear from the trajectories that the pursuers gradually minimize the evader's distance while honoring the pre- defined limits for control. The methodical approach guarantees that Sequential matching of each coordinate ensures capture. in limited time. This illustration backs up the theoretical re- results of the paper, offering a perceptive comprehension of the developed pursuer tactics with nth-order dynamics.

FUTURE RESEARCH DIRECTIONS

Several possible extensions of the present study can be considered as follows.

- Investigate similar pursuit-evasion differential games with higher-order dynamics in Euclidean spaces of arbitrary dimension.
- Another interesting line of research is the introduction of pursuit-evasion differential games involving fractional-order dynamics, both in the plane and Euclidean spaces.

DATA AVAILABILITY

The data will be available on request from the corresponding author.

ACKNOWLEDGMENT

We appreciate the helpful remarks, observations, and recommendations provided by the article's reviewers. Without a doubt, The article's quality was enhanced by the reviewers' reports.

References

[1] G. Ibragimov, "Game problem on a closed convex set", Matematicheskie Trudy **4** (2001) 96. https://www.mathnet.ru/eng/mt14.

[2] G. Ibragimov, P. Karapanan & A. A. Idham, "Pursuit differential game of two pursuers and one evader with coordinate-wise integral constraints in $\mathbb{R}^2$", in *Proc. Int. Conf. on Research and Education in Mathematics*, Kuala Lumpur, Malaysia, 2015, pp. 223–226. https://doi.org/10.1109/ICREM.2015.7357058.

[3] G. Ibragimov, S. Yunusa & A. A. Idham, "Effective pursuit strategies in simple motion differential game of three pursuers and one evader", International Journal of Pure and Applied Mathematics **111** (2016) 409. https://doi.org/10.12732/ijpam.v111i3.6.

[4] A. A. Idham, G. Ibragimov, S. K. Atamurat & I. S. Akamal, "Differential game with many pursuers when controls are subject to coordinate-wise integral constraints", Malaysian Journal of Mathematical Sciences **10** (2016) 195. http://psasir.upm.edu.my/id/eprint/52341/.

[5] I. A. Alias, G. Ibragimov, F. Massimiliano, M. Salimi & M. Monsi, "Differential game of many pursuers with integral constraints on a convex set in the plane", arXiv preprint arXiv:1505.00054, 2015. https://doi.org/10.48550/arXiv.1505.00054.

[6] A. J. Badakaya & M. A. Umar, "An evasion differential game problem on the plane", Bayero Journal of Pure and Applied Sciences **11** (2018) 229. http://dx.doi.org/10.4314/bajopas.v11i2.31.

[7] F. Massimiliano, G. Ibragimov, A. A. Idham & M. Salimi, "Pursuit differential game of many pursuers with integral constraints on a compact convex set", Bulletin of the Malaysian Mathematical Sciences Society **43** (2020) 2929. https://doi.org/10.1007/s40840-019-00844-3.

[8] P. Karapanan, G. Ibragimov & A. A. Alias, "Guaranteed result for the differential game of two pursuers and one evader on a cylinder", Malaysian Journal of Mathematical Sciences **15** (2021) 61. http://psasir.upm.edu.my/id/eprint/97488.

[9] C. Pierre, *Introduction to differential games*, Ph.D. dissertation, Université de Brest, Brest, France, 2010. https://www.ceremade.dauphine.fr/~cardaliaguet/CoursJeuxDiff.pdf.

[10] A. J. Badakaya, H. Abdullahi & M. Salimi, "Pursuit game in a closed convex set on a Euclidean space", Differential Equations and Dynamical Systems **32** (2024) 1215. https://doi.org/10.1007/S12591-022-00621-y.

[11] B. Muhammad & H. Abdullahi, "Differential game with integral constraints in a Hilbert space", International Journal of Research and Innovation in Applied Science **10** (2025) 155. https://doi.org/10.51584/IJRIAS.2025.100500014.

[12] M. U. Bashir, H. Abdullahi, A. Yahaya & S. M. Tsoho, "Many pursuers and one evader game", International Journal of Research and Innovation in Applied Science **10** (2025) 712. https://doi.org/10.51584/IJRIAS.2025.10040060.

[13] G. Ibragimov & M. Salimi, "Pursuit-evasion differential game with many inertial players", Mathematical Problems in Engineering **2009** (2009) 653723. https://doi.org/10.1155/2009/653723.