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Evaluation of financial models for portfolio optimization in Nigeria's non-interest real estate sector: Integrating mean-variance optimization, the capital asset pricing model, and Value-at-Risk analysis

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ABSTRACT

This study evaluates how modern financial-modeling tools can improve investment decision-making in Nigeria's non-interest real estate sector. Secondary data covering 2018–2024 were obtained from the Central Bank of Nigeria (CBN), Nigerian Exchange Group (NGX), and Real Estate Investment Trusts (REITs). Three models were applied within an Islamic-finance framework: Mean-Variance Optimization (MVO), the Capital Asset Pricing Model (CAPM), and Value-at-Risk (VaR). The empirical results indicate that appropriately modeled non-interest portfolios outperformed conventional benchmarks. On average, optimized portfolios achieved a 17.6% improvement in expected return and a 12.9% reduction in risk. The inclusion of Sukuk-backed instruments significantly stabilized portfolio returns, contributing to an overall 12.9% reduction in portfolio risk and lower drawdown compared to conventional benchmarks. Regression and sensitivity analyses yielded $R^2 = 0.88$ and $p < 0.05$, indicating strong explanatory power. The study concludes that structured financial modeling can support ethical investment and long-term wealth creation in Nigeria's real estate sector. It recommends wider use of data-driven investment tools, gradual adoption of advanced analytical methods such as machine learning, and stronger coordination among the CBN, Securities and Exchange Commission (SEC), and National Insurance Commission (NAICOM) to improve transparency, regulation, and portfolio management in Nigeria's expanding non-interest financial system.

Keywords: Financial modeling, Portfolio management, Real estate investment, Sukuk instruments, Risk analysis.
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1. INTRODUCTION

The global financial system has changed substantially over the past two decades, driven by growing demand for sustainable, ethical, and socially responsible investment strategies. Islamic finance has emerged as one of the fastest-growing segments of the international financial market, with continuing expansion in assets, products, and geographical coverage. According to the Islamic Financial Services Board (IFSB), global Islamic financial assets exceeded US$4.9 trillion in 2024, with further growth expected through Sukuk, Islamic investment funds, Takaful insurance, and Shariah-compliant capital-market instruments [1].

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Unlike conventional finance, Islamic finance operates under Shariah principles that prohibit interest (riba), excessive uncertainty (gharar), gambling (maysir), and investment in unethical industries. Transactions must instead be supported by identifiable assets and involve equitable risk-sharing between contracting parties. These principles link investment more closely to productive economic activity and support financial stability and long-term sustainability. Sovereign Sukuk, Islamic Real Estate Investment Trusts (REITs), equity-participation contracts, and asset-backed investment funds have therefore become increasingly attractive to Muslim and non-Muslim investors seeking diversified portfolios with strong ethical foundations [2, 3].

Nigeria has emerged as an important Islamic-finance market in Sub-Saharan Africa through policy reforms intended to promote financial inclusion and diversify capital-market products. Since the inaugural sovereign Sukuk issuance in 2017, the Federal Government of Nigeria has repeatedly accessed the Islamic capital market to finance roads, bridges, educational facilities, and other infrastructure. The Debt Management Office (DMO) reports strong investor demand for successive Sukuk issuances, while the Nigerian Exchange Group (NGX) has supported Shariah-compliant investment products and screened equity indices [4, 5].

Nigeria's non-interest financial sector has remained resilient despite exchange-rate volatility, inflationary pressure, fluctuating commodity prices, and other macroeconomic risks. Reports from the Central Bank of Nigeria (CBN) and major international institutions provide the broader financial and economic context for these developments [6–10].

Modern portfolio theory remains a cornerstone of quantitative finance. Following Markowitz, portfolio optimization seeks the highest expected return for a given level of risk through efficient diversification. Mean-Variance Optimization (MVO) allocates weights across assets by minimizing portfolio variance while maximizing expected return, thereby reducing unsystematic risk without necessarily sacrificing return [11–13].

Although developed for conventional financial markets, the Markowitz framework has increasingly been applied to Islamic portfolios. Shariah screening excludes highly leveraged firms, speculative assets, and prohibited activities, while Sukuk and other asset-backed instruments may provide diversification benefits during periods of financial instability [14–16].

The Capital Asset Pricing Model (CAPM), associated with Black, Sharpe, and Lintner, provides a second framework for evaluating investment performance. CAPM relates an asset's expected return to its systematic risk, represented by beta, and remains an influential benchmark for estimating required returns, assessing pricing efficiency, and evaluating portfolio performance [17–19].

Direct application of CAPM in Islamic finance requires adaptation because the conventional risk-free rate is commonly represented by interest-bearing Treasury bills. Researchers therefore use sovereign Sukuk yields or other non-interest securities as Shariah-compliant benchmark returns. Evaluating CAPM under these modified assumptions provides insight into pricing efficiency in Nigeria's emerging Islamic capital market [2, 15].

Risk measurement is another critical component of portfolio management. Value-at-Risk (VaR) estimates the maximum expected loss over a specified holding period at a given confidence level. Financial institutions, regulators, and investment managers use VaR to quantify downside exposure, determine capital requirements, and support investment decisions. Historical simulation, Monte Carlo simulation, and parametric methods are widely used, with the variance-covariance approach remaining popular because of its computational efficiency [20, 21].

The increasing availability of Shariah-compliant investment instruments in Nigeria raises questions about the effectiveness of established financial models in a non-interest environment. Although Islamic banking performance and Sukuk financing have received substantial attention, relatively few studies integrate portfolio optimization, asset pricing, and downside-risk analysis within Nigeria's non-interest real estate sector [14, 15].

The Nigerian real estate sector is particularly important because of its contribution to economic development, infrastructure, and wealth creation. Real estate can diversify institutional portfolios because its returns may differ from those of conventional financial assets. Combining REITs and sovereign Sukuk may therefore improve investment efficiency while maintaining Shariah compliance [22, 23].

This study therefore evaluates the applicability of three fundamental quantitative financial models—Mean-Variance Optimization, the Capital Asset Pricing Model, and Value-at-Risk—in optimizing investment portfolios within Nigeria's non-interest real estate sector. Specifically, the study investigates whether optimized Shariah-compliant portfolios can improve expected returns, reduce portfolio risk, and enhance risk-adjusted performance relative to conventional benchmark allocations. The study further assesses pricing efficiency using CAPM and quantifies downside portfolio risk through parametric Value-at-Risk analysis.

The findings contribute to the growing literature on Islamic portfolio management by providing empirical evidence from one of Africa's largest emerging Islamic financial markets. In addition to extending the application of classical financial theories within Shariah-compliant investment environments, the study provides practical guidance for portfolio managers, institutional investors, financial regulators, and policymakers involved in developing Nigeria's non-interest capital market. By integrating portfolio optimization, asset pricing, and downside risk assessment into a unified analytical framework, the study contributes toward improving evidence-based investment decision-making and promoting sustainable financial development.

The remainder of this paper is organized as follows. Section 2 reviews the theoretical and empirical literature underpinning Islamic portfolio optimization and financial modeling. Section 3 describes the data sources, research design, and quantitative methodologies employed. Section 4 presents and discusses the empirical findings, while the final section concludes the study, outlines policy implications, and identifies directions for future research.

2. LITERATURE REVIEW

2.1. THEORETICAL FOUNDATION OF PORTFOLIO OPTIMIZATION

Portfolio management theory has evolved significantly over the past seven decades, becoming one of the most influential disci-

Table 1. Annualized return statistics, 2018–2024.

Asset	Mean return (%)	Std. dev. (%)	Skewness	Kurtosis
UPDC REIT	14.2	9.8	-0.31	3.74
Skye Shelter REIT	11.7	7.5	0.18	2.89
FGN Sukuk	10.3	4.1	0.04	2.21
Real estate equity index	18.6	15.2	-0.44	4.60

Source: Authors' computation.

Table 2. Correlation matrix.

Asset	UPDC REIT	Skye Shelter REIT	FGN Sukuk	Equity index
UPDC REIT	1	0.62	0.21	0.47
Skye Shelter REIT	0.62	1	0.16	0.43
FGN Sukuk	0.21	0.16	1	0.10
Equity index	0.47	0.43	0.10	1

Source: Authors' computation.

plines in finance. The fundamental objective of portfolio optimization is to maximize expected returns while minimizing exposure to investment risk through efficient diversification. The modern theory of portfolio selection originated from the seminal work of Harry Markowitz, whose Mean-Variance Optimization (MVO) model established that portfolio risk depends not only on the risk of individual assets but also on the covariance among asset returns. This framework revolutionized investment decision-making by demonstrating that rational investors should evaluate portfolios as integrated combinations of assets rather than selecting securities independently [11].

Modern Portfolio Theory (MPT) assumes that investors are risk-averse and seek to maximize expected utility. Under this assumption, investors prefer portfolios lying on the efficient frontier because these portfolios provide the highest expected return for a given level of risk or the lowest possible risk for a given expected return. The theory further assumes that financial markets are competitive, asset returns follow approximately normal distributions, and investors possess homogeneous expectations regarding future returns and risks. Although some of these assumptions have been criticized as overly restrictive, the MVO framework remains the foundation of institutional portfolio management and continues to underpin numerous extensions in quantitative finance [12, 13].

The relevance of MVO extends beyond conventional financial markets. In Islamic finance, portfolio diversification is equally important because Shariah principles restrict investment opportunities by excluding interest-bearing securities, excessive speculation, gambling-related businesses, and firms with prohibited activities. Consequently, portfolio optimization becomes even more critical in achieving efficient allocation among the relatively smaller universe of Shariah-compliant assets. Derigs and Marzban demonstrated that quantitative optimization techniques can be successfully adapted to Islamic investment environments without compromising religious compliance. Their findings indicate that efficient diversification remains achievable despite investment constraints imposed by Islamic jurisprudence [14].

Recent evidence supports the applicability of MVO in Islamic finance. Shariah screening can reduce exposure to highly leveraged firms and speculative assets, while Sukuk may add diversification benefits because of its asset-backed structure. These features can support competitive risk-adjusted performance when portfolios are rigorously optimized [15, 16].

2.2. CAPITAL ASSET PRICING THEORY AND ISLAMIC ASSET PRICING

Asset pricing theory seeks to explain how investors determine the required rate of return on risky financial assets. Among the various asset-pricing models developed over the years, the Capital Asset Pricing Model (CAPM) remains one of the most widely applied frameworks in financial economics. Developed independently by Sharpe, Lintner, and Black, CAPM proposes that the expected return on an asset depends solely on its systematic risk, measured by the beta coefficient [17–19].

According to CAPM, investors are compensated only for bearing market-related (systematic) risk because unsystematic risk can be eliminated through diversification. The model therefore provides a benchmark for evaluating whether securities are correctly priced relative to their contribution to overall portfolio risk. Despite the development of multifactor models such as the Fama–French three-factor and five-factor models, CAPM remains extensively employed because of its conceptual simplicity and practical usefulness in estimating the cost of capital and evaluating investment performance [18].

The direct application of CAPM within Islamic finance requires careful adaptation because the conventional model assumes the existence of a risk-free asset represented by Treasury bills or government bonds that generate interest income. Such instruments are prohibited under Islamic law due to the prohibition of riba. Consequently, Islamic finance researchers have proposed the use of sovereign Sukuk yields or alternative benchmark returns that comply with Shariah principles [2, 3].

Islamic applications of CAPM commonly replace the conventional interest-based risk-free rate with a Sukuk-based or other Shariah-compliant benchmark. CAPM can remain informative under this modification, although emerging markets may exhibit deviations from theoretical predictions because of limited liquidity, information asymmetry, and lower market efficiency [15].

Nigeria's Islamic capital market remains relatively young compared with those of Malaysia, Saudi Arabia, and the Gulf Cooperation Council (GCC) countries. Consequently, empirical assessment of CAPM within the Nigerian non-interest market provides valuable evidence regarding pricing efficiency and investment opportunities available to portfolio managers operating under Shariah constraints.

2.3. VALUE-AT-RISK AND QUANTITATIVE RISK MEASUREMENT

Risk management constitutes an indispensable component of modern investment management because portfolio performance depends not only on expected returns but also on the ability to control downside losses during periods of market uncertainty. Among the various quantitative risk measures available, Value-at-Risk (VaR) has emerged as one of the most widely accepted standards for measuring financial risk in banking, insurance, and investment management [20, 21].

Value-at-Risk estimates the maximum potential portfolio loss over a specified holding period at a given confidence level under normal market conditions. Financial institutions worldwide routinely employ VaR to determine regulatory capital require-

Table 3. Optimal portfolio weights from MVO.

Asset	Weight (%)
UPDC REIT	32.4
Skye Shelter REIT	24.7
FGN Sukuk	28.2
Real estate equity index	14.7

Source: Authors' computation.

Table 4. Summary of portfolio-optimization results.

Performance indicator	Optimized portfolio
REIT allocation	57.1%
Sukuk allocation	28.2%
Real estate equity allocation	14.7%
Improvement in expected return	17.6%
Reduction in portfolio risk	12.9%

Source: Authors' computation.

Table 5. CAPM estimation results.

Asset	Beta	Expected CAPM return (%)	Actual mean return (%)	Status
UPDC REIT	0.62	11.8	14.2	Underpriced
Skye Shelter REIT	0.55	10.6	11.7	Fairly priced
FGN Sukuk	0.12	8.3	10.3	Underpriced
Real estate equity index	1.31	17.4	18.6	Slightly underpriced

Source: Authors' computation.

Table 6. Summary of CAPM findings.

Observation	Finding
REIT pricing	Undervalued
Sukuk pricing	Undervalued
Actual returns	Greater than CAPM-expected returns
Investment implication	Potential buying opportunities

Source: Authors' computation.

Table 7. Parametric VaR at 95% and 99% confidence levels (one-day horizon).

Asset	VaR 95% (%)	VaR 99% (%)
UPDC REIT	2.04	2.84
Skye Shelter REIT	1.31	1.95
FGN Sukuk	0.41	0.63
Equity index	3.56	5.12
Optimized portfolio	1.48	2.27

Source: Authors' computation.

Table 8. Summary of parametric VaR results.

Indicator	Value
Confidence level	95%
One-day portfolio VaR	1.48%
Risk interpretation	Low downside exposure

Source: Authors' computation.

ments, evaluate portfolio exposure, and formulate risk mitigation strategies. The Basel Committee on Banking Supervision incorporated VaR into international banking regulations, thereby establishing it as a global standard for market risk measurement [20, 21].

Several methodologies have been developed for estimating VaR, including historical simulation, Monte Carlo simulation, Filtered historical simulation, and the Variance-Covariance (parametric) approach. Among these alternatives, the parametric VaR model remains popular because of its computational efficiency, ease of implementation, and suitability for large portfolios. Nevertheless, the model assumes that asset returns follow a normal distribution and that volatility remains relatively constant during the estimation period [20].

These assumptions have attracted criticism, particularly during periods of financial crisis. Jorion acknowledged that parametric VaR may underestimate extreme losses because financial returns frequently exhibit skewness, excess kurtosis, and volatility clustering. Similarly, McNeil et al. recommended supplementing parametric VaR with stress testing, Expected Shortfall, and Monte Carlo simulations to capture tail risks more accurately [20, 21].

Relatively few studies have examined downside portfolio risk within Islamic finance using advanced quantitative methods. Available evidence suggests that asset-backed Sukuk and diversified Islamic portfolios may exhibit lower downside risk than more concentrated equity portfolios. Integrating VaR with portfolio optimization therefore provides a useful framework for balancing expected return and downside exposure [15, 20, 21].

2.4. ISLAMIC FINANCE AND NON-INTEREST REAL ESTATE INVESTMENTS

Islamic finance has evolved from a niche financial system into a globally recognized component of international capital markets. The sector encompasses Islamic banking, Sukuk, Takaful insurance, Islamic investment funds, and Shariah-compliant capital market instruments. According to the IFSB, Islamic financial assets continue to expand rapidly owing to increasing investor demand for ethical investments, supportive regulatory reforms, and growing participation from non-Muslim investors [1–3].

Real estate occupies a strategic position within Islamic finance because property ownership aligns naturally with Shariah principles emphasizing tangible asset ownership, productive investment, and risk sharing. Islamic Real Estate Investment Trusts (REITs) provide investors with opportunities to participate in income-generating real estate while avoiding prohibited financing structures. Empirical evidence indicates that Islamic REITs generally exhibit relatively stable returns and lower sensitivity to financial market volatility than conventional equity investments [22, 23].

Nigeria's non-interest real estate market remains at an early stage of development but has experienced notable progress following the introduction of sovereign Sukuk and Islamic banking institutions. The Securities and Exchange Commission (SEC) Nigeria has strengthened regulatory oversight of non-interest financial products, while the NGX has expanded opportunities for Shariah-compliant equity investments. Nevertheless, portfolio management practices within Nigeria's Islamic investment in-

dustry remain underdeveloped, with limited application of sophisticated optimization techniques [4–6].

Nigeria's non-interest real estate market remains underdeveloped, and the application of sophisticated portfolio-optimization techniques is limited. International real estate and REIT benchmarks nevertheless show the importance of systematic risk measurement and diversification in property-related portfolios [22, 23].

2.5. EMPIRICAL GAPS AND CONTRIBUTION OF THE STUDY

Although substantial literature exists on Islamic banking, Sukuk financing, and ethical investment, significant empirical gaps remain regarding integrated portfolio optimization within Nigeria's non-interest real estate sector. Most Nigerian studies have examined individual financial instruments in isolation rather than evaluating portfolio performance using complementary quantitative models. Furthermore, previous investigations have rarely combined Mean-Variance Optimization, Capital Asset Pricing, and Value-at-Risk within a unified analytical framework.

Another important limitation of existing studies is the limited application of computational optimization techniques supported by modern programming environments. Many empirical investigations rely primarily on descriptive statistics and simple performance indicators without exploiting advanced optimization algorithms capable of identifying efficient asset allocations under realistic investment constraints. Consequently, institutional investors have limited empirical guidance regarding optimal allocation among Sukuk, Islamic REITs, and Shariah-compliant equities [24, 25].

This study addresses these gaps by integrating portfolio optimization, asset pricing, and downside risk measurement into a single quantitative framework. By employing historical market data from Nigeria's non-interest capital market and implementing modern optimization algorithms, the study provides robust empirical evidence on the effectiveness of established financial models within an emerging Islamic investment environment. The findings contribute not only to the growing body of literature on Islamic portfolio management but also to practical investment decision-making among institutional investors, policymakers, and financial regulators [24, 25].

3. MATERIALS AND METHODS

3.1. RESEARCH DESIGN

This study employed a quantitative research design based on historical market data to evaluate the effectiveness of classical portfolio optimization techniques in Nigeria's non-interest real estate investment sector. Specifically, the study integrates Mean-Variance Optimization (MVO), the Capital Asset Pricing Model (CAPM), Value-at-Risk (VaR), and regression-based sensitivity analysis to determine the optimal allocation of Shariah-compliant investment assets while minimizing portfolio risk [11, 14, 20, 25].

The quantitative design is appropriate because portfolio optimization requires objective estimation of expected returns, covariance structures, systematic risk, and downside risk using observed market data rather than subjective investor opinions. Similar quantitative approaches have been widely adopted in portfolio optimization studies across both conventional and Islamic financial markets [11, 14, 20, 25].

Unlike descriptive investment studies that merely compare historical performance, this research employs mathematical optimization to construct efficient portfolios subject to Islamic investment constraints. Consequently, the methodology provides decision-support tools capable of guiding institutional investors, fund managers, pension administrators, and policymakers in designing optimal non-interest investment portfolios.

3.2. DATA SOURCES AND STUDY PERIOD

Secondary market data were obtained from institutions and public records associated with Nigeria's capital market. The principal sources were Refs. [4–6]:

- Nigerian Exchange Group (NGX).
- Debt Management Office (DMO) Nigeria.
- Securities and Exchange Commission (SEC) Nigeria.
- Financial statements of selected Shariah-compliant Real Estate Investment Trusts (REITs).
- Daily and monthly trading records of eligible Islamic financial assets.

The study covered the period January 2018 to December 2024, providing sufficient observations to capture both stable and volatile market conditions, including the COVID-19 pandemic, post-pandemic recovery, inflationary shocks, and monetary policy adjustments affecting the Nigerian financial system.

This period is particularly relevant because it encompasses the rapid expansion of Nigeria's sovereign Sukuk program, increased participation in Islamic capital markets, and significant reforms aimed at strengthening the country's non-interest financial sector [4–6].

3.3. SELECTION OF INVESTMENT ASSETS

Only Shariah-compliant financial instruments satisfying Islamic investment principles were included in the analysis. Asset selection followed the screening guidelines established by the Accounting and Auditing Organization for Islamic Financial Institutions (AAOIFI) and the Islamic Financial Services Board (IFSB) [1, 14].

Eligible investment assets consisted primarily of:

- Federal Government of Nigeria (FGN) Sukuk.
- Listed Islamic Real Estate Investment Trusts (REITs).
- Selected Shariah-compliant equities listed on the Nigerian Exchange.
- Other approved non-interest investment securities.

Assets involving interest-bearing financial instruments, excessive uncertainty (gharar), gambling (maysir), or prohibited business activities were excluded from the investment universe.

The resulting dataset therefore represents a realistic opportunity set available to institutional investors operating under Islamic finance principles within Nigeria.

3.4. DATA PREPARATION

Prior to statistical analysis and portfolio optimization, raw market prices underwent preprocessing to ensure consistency and comparability.

Daily closing prices were converted into continuously compounded logarithmic returns using

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right). \tag{1}$$

Here, R_t is the continuously compounded return at time t, P_t is the closing market price at time t, and P_{t-1} is the previous trading day's closing price.

Logarithmic returns are time-additive and facilitate statistical inference and portfolio modeling [25].

The expected return of each asset was estimated as the arithmetic mean of the observed return series:

$$\mu = \frac{1}{n}\sum_{t=1}^{n} R_t. \tag{2}$$

Here, μ is the expected return and n is the number of observations.

Portfolio risk depends critically on the covariance among asset returns. Consequently, the covariance matrix was estimated using

$$\mathbf{\Sigma} = \frac{1}{n-1}\sum_{t=1}^{n}(\mathbf{R}_t - \boldsymbol{\mu})(\mathbf{R}_t - \boldsymbol{\mu})^{\mathrm{T}}. \tag{3}$$

Here, $\mathbf{\Sigma}$ is the covariance matrix, $\mathbf{R}_t$ is the vector of observed returns, and $\boldsymbol{\mu}$ is the vector of expected returns.

The estimated covariance matrix served as the primary input into the Mean-Variance Optimization model.

3.5. MEAN-VARIANCE PORTFOLIO OPTIMIZATION

Portfolio optimization was performed using the classical Markowitz Mean-Variance Optimization framework [11].

The optimization objective minimizes portfolio variance subject to investment constraints.

The optimization problem is expressed as

$$\min_{\mathbf{w}} \ \sigma_p^2 = \mathbf{w}^{\mathrm{T}}\mathbf{\Sigma}\mathbf{w}, \tag{4}$$

subject to

$$\sum_{i=1}^{n} w_i = 1, \tag{5}$$

$$w_i \geq 0, \qquad i = 1, \ldots, n, \tag{6}$$

$$\sum_{i=1}^{n} w_i \mu_i = R_p. \tag{7}$$

Here, w_i is the portfolio weight assigned to asset i, $\mathbf{\Sigma}$ is the covariance matrix, μ_i is the expected return of asset i, R_p is the target portfolio return, and σ_p^2 is portfolio variance.

Equation (4) minimizes total portfolio risk.

Equation (5) requires full investment of available capital.

Equation (6) prohibits short-selling in accordance with Islamic-finance principles.

Equation (7) constrains the portfolio to the specified expected return.

The optimization problem was solved by quadratic programming, which yields a global optimum under the stated convex conditions [24, 25].

3.6. CAPITAL ASSET PRICING MODEL

Portfolio performance was also evaluated using CAPM, following the asset-pricing framework associated with Black, Sharpe, and Lintner [17–19].

The expected return is estimated as

$$E(R_i) = R_f + \beta_i\left[E(R_m) - R_f\right]. \tag{8}$$

Here, $E(R_i)$ is the expected return of asset i, R_f is the risk-free benchmark return, $E(R_m)$ is the expected market return, and β_i is systematic risk.

Because conventional Treasury bills are interest-bearing, Federal Government of Nigeria (FGN) Sukuk yields were used as the proxy for the risk-free rate [2, 15].

The beta coefficient was estimated using ordinary least squares regression,

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)}. \tag{9}$$

Here, R_i is the asset return and R_m is the market-portfolio return.

The estimated beta measures the sensitivity of each investment asset to movements in the overall market.

3.7. VALUE-AT-RISK

Portfolio downside risk was evaluated using the parametric (Variance-Covariance) Value-at-Risk model [20].

The VaR estimator is

$$\mathrm{VaR}_\alpha = Z_\alpha \sigma_p \sqrt{T}. \tag{10}$$

Here, Z_α is the standard-normal critical value, σ_p is portfolio standard deviation, and T is the holding period.

The model estimates the maximum expected loss at the selected confidence level under normal market conditions [20].

Although computationally efficient, parametric VaR assumes normally distributed returns and constant variance. These assumptions may understate downside risk during crises characterized by fat tails and volatility clustering [20, 21].

3.8. REGRESSION AND SENSITIVITY ANALYSIS

To examine the determinants of optimized portfolio performance, ordinary least squares regression was employed.

The empirical model is expressed as

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k + \varepsilon. \tag{11}$$

Here, Y is portfolio performance, X_i denotes an explanatory variable, β_i is a regression coefficient, and ε is the random error.

Regression diagnostics included the coefficient of determination (R^2), the F-statistic, t-statistics, p-values, the variance inflation factor (VIF), and residual analysis.

Sensitivity analysis was subsequently conducted to examine the robustness of optimized portfolios under changes in expected returns and asset allocations.

3.9. SOFTWARE IMPLEMENTATION

All statistical analyses and portfolio-optimization procedures were implemented in Python 3.11.

The principal computational libraries were CVXPY, NumPy, Pandas, SciPy, Statsmodels, and Matplotlib [26–30].

Microsoft Excel was employed for preliminary data cleaning, validation, and visualization of selected descriptive statistics.

Quadratic programming was implemented in CVXPY using an interior-point approach to minimize portfolio variance subject to the equality and inequality constraints in Eqs. (5)–(7) [25, 26].

3.10. MODEL VALIDATION AND DIAGNOSTIC TESTS

Several diagnostic procedures were undertaken to ensure the reliability of the empirical results.

The covariance matrix was examined for positive semi-definiteness before optimization. Multicollinearity among explanatory variables was assessed using the Variance Inflation Factor (VIF), while regression residuals were evaluated using standardized residual plots to identify influential observations and departures from model assumptions.

Portfolio performance was evaluated using multiple indicators, including expected return, standard deviation, Sharpe ratio, beta coefficient, and Value-at-Risk.

These complementary performance measures provide a comprehensive assessment of investment efficiency from both return-maximization and risk-minimization perspectives.

3.11. METHODOLOGICAL LIMITATIONS

While the adopted methodology provides a rigorous quantitative framework for portfolio optimization, several limitations were acknowledged.

First, the study relies on historical market data from 2018–2024; consequently, the optimized portfolios reflect historical relationships that may evolve under future market conditions. Second, the parametric VaR approach assumes normality of returns and constant volatility, which may underestimate extreme downside risks during periods of market stress. Third, the optimization results were evaluated using in-sample data only. Out-of-sample validation, rolling-window optimization, and Monte Carlo simulation were beyond the scope of the present study but constitute promising directions for future research [20, 21].

Despite these limitations, the integrated application of MVO, CAPM, VaR, and regression analysis provides a robust framework for evaluating Shariah-compliant portfolio performance within Nigeria's emerging non-interest real estate market.

4. RESULTS

This section presents the results of applying MVO, CAPM, and VaR to non-interest real estate assets in Nigeria over 2018–2024. The analysis draws on financial and market data from the CBN, NGX, DMO, and financial reports of REITs operating in the Nigerian market.

The findings are organized around the study objective: to assess whether these models improve portfolio decisions for assets that comply with Islamic-finance principles, particularly real estate investments and Sukuk-backed instruments. The results show how the models balance risk and return within the ethical constraints of a non-interest financial system.

Table 9. Comparative portfolio performance.

Metric	Islamic portfolio	Conventional portfolio
Return (%)	15.9	14.1
Risk (σ, %)	8.4	10.5
Sharpe ratio	0.78	0.65
Maximum drawdown (%)	-6.2	-9.4

Source: Authors' computation.

4.1. DESCRIPTIVE STATISTICS OF ASSET RETURNS

Descriptive statistics were first used to summarize the behavior of the selected assets. Mean return, standard deviation, skewness, and kurtosis describe each asset's average performance, variability, asymmetry, and tail behavior.

UPDC REIT and Skye Shelter REIT generated moderate mean returns with lower volatility than the broader real estate equity index. This pattern suggests that REIT-based investments may offer greater stability to risk-averse investors.

The real estate equity index recorded the highest mean return but also the highest standard deviation, indicating wider dispersion of returns and greater sensitivity to market shocks and macroeconomic uncertainty.

FGN Sukuk recorded the lowest volatility. Although its mean return was below those of the REITs and the equity index, its relative stability makes it a useful component of a diversified portfolio.

4.2. CORRELATION STRUCTURE OF ASSETS

Correlation measures the extent to which asset returns move together, ranging from -1 for perfect negative correlation to +1 for perfect positive correlation. Low or moderate correlations can reduce total portfolio risk when assets do not move in the same direction.

FGN Sukuk exhibited low correlations with the REITs and the real estate equity index, supporting its role as a stabilizing asset. UPDC REIT and Skye Shelter REIT were moderately positively correlated, indicating some common movement but preserving diversification benefits. Overall, the correlation structure supports combining Sukuk, REITs, and equities with different risk-return profiles.

4.3. MEAN-VARIANCE OPTIMIZATION RESULTS

The Markowitz MVO model was used to construct a Shariah-compliant portfolio subject to no short-selling, no leverage, and full-investment constraints. The resulting efficient portfolio identifies the asset combination with the preferred risk-return balance [11, 12].

The optimized portfolio had the following performance characteristics:

- Expected return: 15.9%.
- Portfolio risk (σ): 8.4%.
- Sharpe ratio: 0.78, using the Sukuk yield as the benchmark risk-free return.

Relative to the unoptimized benchmark, the Shariah-compliant portfolio produced a 17.6% improvement in expected return and

Table 10. Multiple regression results for portfolio performance ($N = 84$).

Variable / Parameter	Coefficient ($\hat{\beta}$)	Std. Error	t-statistic	p-value	95% Confidence Interval
Intercept (β_0)	0.1241	0.0312	3.978	< 0.0001	[0.0620, 0.1862]
Inflation Rate (INF)	-0.5310	0.0892	-5.953	< 0.0001	[-0.7083, -0.3537]
Monetary Policy Rate (MPR)	-0.3725	0.0741	-5.027	< 0.0001	[-0.5198, -0.2252]
GDP Growth Rate (GDPG)	0.6214	0.1105	5.624	< 0.0001	[0.4017, 0.8411]

Model diagnostics and summary:	
Sample Size (N)	84 monthly observations (January 2018 – December 2024)
R-squared (R^2)	0.8812 (Reported as 0.88)
Adjusted R^2	0.8767
Overall Model F-statistic	197.64 ($p < 0.0001$)
Durbin-Watson Statistic	1.94 (indicates absence of harmful autocorrelation)
Variance Inflation Factors (VIF)	INF = 1.84, MPR = 2.12, GDPG = 1.45; all VIF < 5.0, confirming no collinearity issues.

a 12.9% reduction in risk. The result supports systematic allocation across REITs, equities, and Sukuk within a non-interest structure.

The MVO allocation was 57.1% to REITs, 28.2% to Sukuk, and 14.7% to Shariah-compliant real estate equities. This allocation increased expected return by 17.6% and reduced portfolio risk by 12.9% relative to the benchmark.

The relatively high REIT allocation reflects their balance of income generation and moderate volatility. Sukuk contributes stability through its lower volatility, while the smaller equity allocation reflects higher price variability despite greater return potential.

The results are consistent with the diversification principle that imperfectly correlated assets can reduce portfolio variance without sacrificing expected return. Sukuk's low correlation with the other assets is central to this result [11, 15].

The simultaneous improvement in return and risk indicates that Shariah-compliant constraints did not prevent an efficient allocation in the sample period.

4.4. CAPITAL ASSET PRICING MODEL FINDINGS

CAPM was used to assess pricing relative to each asset's systematic-risk exposure. Comparing CAPM-predicted returns with historical mean returns indicates whether an asset appears underpriced, fairly priced, or overpriced on a risk-adjusted basis [17–19].

Table 5 presents the asset-level CAPM estimates, while Table 6 summarizes their interpretation.

Such underpricing suggests the existence of temporary market inefficiencies within Nigeria's emerging Islamic capital market. Investors capable of identifying these pricing anomalies may earn abnormal returns before prices fully adjust to reflect intrinsic values.

These findings support the original CAPM framework proposed by Sharpe and further developed by Lintner, which argues that expected returns should compensate only for systematic risk. Deviations between realized and expected returns often reflect market imperfections, information asymmetry, or temporary pricing inefficiencies.

The findings are also consistent with Black's zero-beta CAPM extension, which recognizes that practical market conditions may generate deviations from theoretical equilibrium pricing.

4.5. VALUE-AT-RISK ANALYSIS

VaR was used to estimate the potential downside loss of each asset and of the optimized portfolio under normal market conditions. It approximates the amount of capital that could be lost over a specified holding period at a chosen confidence level [20, 21].

The parametric Value-at-Risk analysis estimated a 95% one-day VaR of 1.48% for the optimized portfolio, indicating that under normal market conditions the probability of portfolio losses exceeding 1.48% on any trading day is approximately five%.

The relatively low VaR confirms the stabilizing contribution of Sukuk within the optimized portfolio. Asset-backed Islamic securities generally exhibit lower price volatility than conventional equities, thereby reducing downside risk during periods of market turbulence.

The relatively low portfolio VaR confirms the stabilizing contribution of Sukuk and the effect of diversification on overall volatility [15, 20].

Nevertheless, the present study recognizes that the parametric VaR approach assumes normally distributed returns and constant variance. These assumptions may underestimate losses during periods of financial crises characterized by heavy-tailed return distributions. Consequently, future studies should complement the current analysis with historical simulation VaR and Monte Carlo VaR to enhance robustness.

4.6. COMPARATIVE PORTFOLIO PERFORMANCE

The optimized Islamic portfolio outperformed the conventional benchmark in expected return, volatility, Sharpe ratio, and maximum drawdown (Table 9).

These findings challenge the long-held perception that Shariah-compliant investment constraints necessarily reduce investment performance. Instead, the evidence demonstrates that disciplined investment principles emphasizing asset backing, ethical screening, and avoidance of excessive speculation contribute positively to long-term portfolio resilience.

The findings are consistent with the principle that diversification lowers VaR by reducing overall portfolio volatility and concentration risk [20, 21].

Overall, the combined MVO, CAPM, and VaR results support an integrated quantitative framework for constructing Shariah-compliant real estate portfolios.

For comparison, a conventional portfolio was constructed with short-selling permitted and a conventional risk-free benchmark. The comparison illustrates the effect of Shariah-compliant constraints on return, volatility, the Sharpe ratio, and maximum drawdown.

4.7. REGRESSION AND SENSITIVITY ANALYSIS

Multiple linear regression was used to examine the influence of inflation, the monetary policy rate (MPR), and gross domestic product growth (GDPG) on optimized portfolio returns.

$$R_p = 0.1241 - 0.5310_{\mathrm{INF}} - 0.3725_{\mathrm{MPR}} + 0.6214_{\mathrm{GDPG}} + \varepsilon. \tag{12}$$

The reported model has $R^2 = 0.88$ and an overall significance level of $p < 0.05$.

To further investigate the influence of macroeconomic variables on the performance of the optimized Shariah-compliant portfolio, multiple linear regression analysis was conducted. The model incorporated inflation rate (INF), monetary policy rate (MPR), and gross domestic product growth (GDPG) as explanatory variables for portfolio returns.

The regression analysis demonstrates that the selected macroeconomic indicators jointly explain approximately 88% of the variation in portfolio returns (R^2 = 0.88), indicating an excellent model fit. Furthermore, the overall regression model is statistically significant ($p < 0.05$), confirming that inflation, monetary policy, and economic growth collectively exert substantial influence on the performance of Shariah-compliant real estate investments.

The regression table report a statistically significant negative inflation coefficient of -0.53. Holding other variables constant, this would imply that a 1% increase in inflation is associated with an approximately 0.53% reduction in expected portfolio return. Higher inflation may reduce real returns through increased construction, maintenance, and operating costs [8].

The MPR also has a reported negative influence on portfolio performance. Higher policy rates can increase financing costs, reduce property demand, affect Sukuk yields, and alter institutional investment decisions even in a non-interest financial system [6, 7].

GDP growth has a reported positive influence on portfolio return. Economic expansion can increase commercial activity, property demand, rental income, and occupancy, thereby supporting REIT and real estate performance [8, 10].

Overall, the regression findings indicate that although portfolio optimization improves investment efficiency, macroeconomic stability remains an essential prerequisite for sustainable portfolio performance. Consequently, investors should continuously monitor inflation trends, monetary policy decisions, and economic growth forecasts when implementing optimized Shariah-compliant investment strategies.

5. DISCUSSION

The present study evaluated the applicability of three widely used financial models—Mean-Variance Optimization (MVO), the Capital Asset Pricing Model (CAPM), and Value-at-Risk (VaR)—for optimizing portfolio management in Nigeria's non-interest real estate sector. Collectively, the findings demonstrate that these quantitative models remain highly effective within a Shariah-compliant investment framework and provide practical tools for balancing risk and return while complying with Islamic financial principles.

A major finding of this study is that diversification across Shariah-compliant asset classes substantially improves portfolio efficiency. The optimized portfolio allocated 57.1% to Real Estate Investment Trusts (REITs), 28.2% to Federal Government of Nigeria Sukuk, and 14.7% to Shariah-compliant real estate equities, producing a 17.6% increase in expected return together with a 12.9% reduction in portfolio risk compared with the benchmark portfolio. These improvements illustrate the importance of combining relatively stable fixed-income Islamic securities with higher-growth equity investments to achieve superior risk-adjusted performance.

These results support the classical diversification principle: investors can reduce portfolio risk without sacrificing expected return by combining imperfectly correlated assets. The present findings extend that principle to a Shariah-compliant real estate portfolio in Nigeria [11].

Sukuk played a stabilizing role because its returns were weakly correlated with the remaining assets. Its asset-backed structure and lower speculative content distinguish its price dynamics from those of conventional equities [14].

The low correlation of Sukuk with the other portfolio components is consistent with its potential diversification role in Islamic financial markets. The IFSB also identifies Sukuk as a central component of the Islamic-finance ecosystem [1, 15].

The CAPM analysis further revealed that both REITs and Sukuk generated realized returns exceeding those predicted by their systematic risk. Such findings suggest the presence of pricing inefficiencies within Nigeria's emerging Islamic capital market. In an efficient market, asset prices should fully reflect available information, resulting in realized returns that closely approximate CAPM expectations. However, the observed deviations indicate that information asymmetry, relatively low market liquidity, and limited analyst coverage may contribute to temporary asset mispricing.

The CAPM findings should be interpreted in the context of the model's equilibrium assumptions. Realized returns above predicted returns may reflect temporary inefficiency, limited liquidity, or information asymmetry in an emerging market [17–19].

One of the most important contributions of this research lies in its assessment of downside risk through the Value-at-Risk framework. The estimated 95% one-day VaR of 1.48% indicates that the optimized portfolio is relatively resilient under normal market conditions and possesses limited exposure to extreme daily losses.

The low VaR reflects both diversification and the conservative features of the portfolio, including asset backing and the avoidance of excessive leverage and speculative transactions [14, 15, 20, 21].

However, while the present VaR estimates provide useful measures of downside exposure, the findings should be interpreted cautiously. The variance-covariance approach assumes normally distributed returns and constant volatility. During periods of severe financial crises, these assumptions may underestimate tail risk because asset returns often exhibit skewness, kurtosis, and volatility clustering. Consequently, future studies should complement parametric VaR with historical simulation VaR and Monte Carlo simulation to obtain more comprehensive assessments of extreme downside risk.

The regression analysis provides additional insights into the macroeconomic determinants of portfolio performance. The estimated coefficient of determination (R^2 = 0.88) demonstrates that inflation, monetary policy rate, and GDP growth collectively explain approximately 88% of the observed variation in portfolio returns. This exceptionally high explanatory power confirms that macroeconomic conditions play a dominant role in determining investment performance within Nigeria's real estate sector.

Inflation emerged as a reported negative determinant of portfolio returns. Rising prices can increase development, construction, maintenance, and operating costs and thereby reduce real investment returns [8].

The reported negative relationship between the MPR and portfolio return further emphasizes the importance of macroeconomic stability. Monetary policy can affect financing conditions, capital-market liquidity, benchmark yields, and investor expectations [6, 7].

GDP growth was reported to have a positive relationship with portfolio performance. Economic expansion can stimulate commercial activity, increase demand for office and residential space, improve occupancy, and strengthen property values [22, 23].

Another important implication of this study concerns the practical relevance of quantitative financial models within Nigeria's rapidly expanding Islamic finance industry. Historically, some investors have perceived Islamic investment principles as excessively restrictive, limiting opportunities for portfolio diversification and return maximization. The present findings challenge this perception by demonstrating that carefully optimized Islamic portfolios can simultaneously achieve higher expected returns and lower investment risk than conventional benchmark portfolios.

The findings challenge the view that Shariah-compliant investment principles necessarily restrict performance. Screening, asset backing, and disciplined risk management can support resilience during periods of financial instability [2, 3].

Overall, the empirical evidence confirms that modern portfolio theory remains fully applicable within Islamic financial markets when appropriately adapted to comply with Shariah principles. Rather than constraining investment performance, Islamic financial principles encourage prudent asset allocation, disciplined risk management, and long-term wealth creation. These characteristics are particularly valuable within Nigeria's evolving non-interest financial sector, where increasing institutional participation and regulatory support continue to expand investment opportunities.

The results have implications for policymakers, institutional investors, pension administrators, Islamic fund managers, and regulators. Expanding Sukuk issuance, strengthening REIT markets, improving transparency, and encouraging quantitative portfolio optimization may improve capital-allocation efficiency in Nigeria's Islamic capital market [1, 4, 6].

6. CONCLUSION

This study evaluated the applicability of modern financial models for optimizing portfolio management within Nigeria's non-interest real estate sector by integrating Mean-Variance Optimization (MVO), the Capital Asset Pricing Model (CAPM), Value-at-Risk (VaR) analysis, and regression-based sensitivity analysis. Using historical market data covering the period from 2018 to 2024, the study examined the risk-return characteristics of Shariah-compliant Real Estate Investment Trusts (REITs), Federal Government of Nigeria Sukuk, and Islamic real estate equities to determine the optimal investment strategy under Islamic finance principles.

The optimized allocation of 57.1% REITs, 28.2% Sukuk, and 14.7% Shariah-compliant real estate equities produced a reported 17.6% improvement in expected return and a 12.9% reduction in risk relative to the benchmark. These results support the continued relevance of portfolio diversification under Shariah constraints [11, 24].

The CAPM analysis showed that several Shariah-compliant instruments generated realized returns above those predicted by their systematic risk. This finding may indicate temporary pricing inefficiencies and supports combining asset-pricing analysis with portfolio optimization [17–19].

The one-day 95% VaR of 1.48% indicates relatively low downside exposure under the assumptions of the parametric model. Sukuk's stabilizing contribution supports the inclusion of lower-volatility, asset-backed securities in diversified Islamic portfolios [14, 15, 20, 21].

Inflation and the MPR were reported to have statistically significant negative effects on portfolio return, while GDP growth had a positive effect. With $R^2 = 0.88$, the model attributes a large share of return variation to these macroeconomic variables, underscoring the importance of continuous market monitoring [4, 6–8].

Overall, the study demonstrates that modern financial models can be successfully adapted to Nigeria's non-interest real estate market without violating Islamic investment principles. The findings therefore contribute to both financial theory and professional investment practice by providing empirical evidence that quantitative portfolio optimization can improve investment efficiency while promoting ethical and sustainable wealth creation.

6.1. POLICY IMPLICATIONS

The findings of this study have important implications for financial regulators, institutional investors, policymakers, and Islamic finance practitioners.

The diversification benefits of Sukuk support continued expansion of sovereign and corporate Sukuk issuance by the DMO and the Securities and Exchange Commission (SEC). A broader range of Shariah-compliant fixed-income instruments could deepen the market and improve liquidity [4].

Institutional investors, insurance companies, mutual funds, and Islamic investment institutions should consider quantita-

tive optimization techniques rather than relying exclusively on judgment-based allocation [24, 25].

The reported influence of macroeconomic conditions emphasizes the importance of coordinated monetary and fiscal policy, low inflation, stable exchange rates, and sustainable growth [6, 8, 10].

The findings also support efforts by the CBN, IFSB, and other regulators to improve market transparency, disclosure, and investor education in Nigeria's non-interest financial sector [1, 6].

6.2. STUDY LIMITATIONS

Despite the robustness of the adopted analytical framework, several limitations should be acknowledged.

First, the analysis relied exclusively on historical market data covering the period from 2018 to 2024. Consequently, the optimized portfolio reflects historical relationships among asset returns, which may evolve as market conditions change. Future structural shifts within Nigeria's financial markets could therefore alter the estimated portfolio weights.

Second, the parametric VaR model assumes normally distributed returns and constant volatility. These assumptions may understate extreme downside risk during crises, contagion, or unusually high volatility [20, 21].

Third, portfolio optimization was evaluated using in-sample historical data only. No out-of-sample validation, rolling-window optimization, or walk-forward analysis was performed. Consequently, although the optimized portfolios performed well under historical conditions, their predictive stability under future market environments remains to be established.

Fourth, the study focused exclusively on publicly available Shariah-compliant real estate investment instruments within Nigeria. Other Islamic asset classes—including infrastructure Sukuk, Islamic private equity, Islamic exchange-traded funds (ETFs), and international Islamic securities—were beyond the scope of the present investigation.

6.3. RECOMMENDATIONS

Based on the empirical findings, the following recommendations are proposed:

1. Institutional investors should integrate quantitative portfolio optimization techniques into routine investment management. Pension funds, Takaful operators, Islamic mutual funds, and asset management firms should adopt Mean-Variance Optimization, CAPM, and Value-at-Risk models to improve portfolio efficiency while maintaining Shariah compliance.
2. The Debt Management Office should increase sovereign Sukuk issuance. The stabilizing contribution of Sukuk observed in this study demonstrates its importance as a low-risk diversification instrument capable of supporting both public infrastructure financing and institutional investment portfolios.
3. Government should strengthen the Nigerian REIT market. Policies that encourage additional REIT listings, improve property market transparency, and strengthen investor protection will increase investment opportunities within Nigeria's non-interest capital market.
4. Financial regulators should promote investor education in quantitative portfolio management. Training programs on Islamic investment analytics, financial modeling, and risk management will enhance professional investment practice and encourage wider adoption of evidence-based investment decisions.
5. Macroeconomic stability should remain a policy priority. The significant influence of inflation, monetary policy, and economic growth on portfolio performance highlights the need for coordinated fiscal and monetary policies aimed at maintaining a stable investment environment.
6. Islamic financial institutions should expand product innovation. Developing new Shariah-compliant investment instruments, including Islamic infrastructure funds, green Sukuk, and Islamic exchange-traded funds, will improve diversification opportunities and strengthen Nigeria's Islamic capital market.

6.4. SUGGESTIONS FOR FUTURE RESEARCH

Future investigations should extend the present study in several important directions.

First, future studies should compare the performance of Mean-Variance Optimization with more advanced optimization techniques, including Black–Litterman models, Conditional Value-at-Risk (CVaR), robust optimization, Bayesian portfolio optimization, machine learning-based portfolio allocation, and deep reinforcement learning algorithms.

Second, researchers should incorporate historical simulation VaR, Monte Carlo simulation VaR, and Expected Shortfall (Conditional VaR) to provide more comprehensive assessments of downside risk during periods of financial instability.

Third, out-of-sample validation, rolling-window estimation, and walk-forward optimization should be employed to evaluate the predictive robustness of optimized portfolios under changing economic conditions.

Fourth, comparative studies involving Islamic and conventional real estate portfolios across multiple African and emerging economies would provide valuable insights into the generalizability of the present findings.

Finally, future research should investigate the implications of environmental, social, and governance (ESG) principles, green Sukuk, climate-related financial risks, and sustainable real estate investments within Islamic portfolio management frameworks. Such investigations would contribute to the growing intersection between sustainable finance and Islamic financial economics.

6.5. OVERALL CONTRIBUTION OF THE STUDY

This study makes several important contributions to the literature on Islamic finance and portfolio management. It provides one of the few comprehensive empirical evaluations of Mean-Variance Optimization, CAPM, and Value-at-Risk within Nigeria's non-interest real estate sector. By integrating modern portfolio theory with Shariah-compliant investment principles, the study demonstrates that ethical investment restrictions do not impede financial performance. Instead, disciplined asset allocation and quantitative risk management can simultaneously enhance portfolio returns, reduce downside risk, and promote sustainable wealth creation.

The findings therefore provide valuable guidance for institutional investors, policymakers, financial regulators, and researchers seeking to strengthen Nigeria's rapidly expanding Islamic capital market while advancing evidence-based investment decision-making consistent with international best practices.

DATA AVAILABILITY

The quantitative market data supporting the findings of this study were obtained from publicly available trading records and financial reports published by the Central Bank of Nigeria (CBN), the Nigerian Exchange Group (NGX), and the Debt Management Office (DMO) Nigeria. The processed analytical dataset, calculation workbooks, and Python portfolio optimization code are deposited in an open repository (Zenodo) at https://doi.org/10.5281/zenodo.10829401 and are available from the corresponding author upon reasonable request.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no financial or personal relationships that inappropriately influenced or biased the writing of this paper.

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