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Proceedings of the Nigerian Society of Physical Sciences

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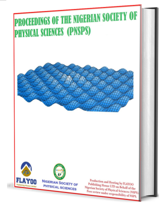

Effect of viscous dissipation on steady-state pressure-driven flow in a horizontal porous channel

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ABSTRACT

This work considers a pressure-driven flow through a horizontal channel. The flow is considered under the influence of viscous dissipation effect and porosity of the channel plates. The equations governing the flow are solved using the method of undetermined coefficients to obtain a closed-form solution for velocity and temperature of the fluid within the system. A simulation of the analytical solutions obtained was carried out on MATLAB and the outcome was presented in graphical form. From the investigation, it can be deduced that viscous dissipation acts to increase fluid temperature.

Keywords: Viscous dissipation, Horizontal channel, Pressure gradient.

DOI:10.61298/pnspsc.2025.2.194

1. INTRODUCTION

Fluid flow within porous channels represents a critical area of study across numerous engineering and environmental domains. The dynamics of fluid movement and heat transfer in these channels are vital for applications in diverse fields, including filtration and purification processes, groundwater management, geological investigations, the petroleum industry, and food processing. In analyzing flows through porous media, it is crucial to consider a range of physical phenomena, particularly viscous dissipation, as it can profoundly influence the behavior of the flow.

Viscous dissipation refers to the irreversible transformation of mechanical energy into thermal energy due to the viscous effects present in a fluid. This phenomenon is crucial for understanding temperature increases, as it represents the conversion of mechanical energy into heat resulting from fluid friction when a fluid traverses a porous channel. Viscous dissipation plays a significant role in influencing fluid flow within porous media and is often an important property that cannot be overlooked. Viscous dissipation is more prominent when the Brinkman number is high and when considering a fluid with very low Prandtl number. The importance of viscous dissipation can be seen in flows in heat exchangers, high speed flow in aerodynamics and flows in lubrication systems [1]. Its implications extend to temperature distribution and the characteristics of pressure-driven flow in porous channels, as well as the efficient functioning of industrial machinery where lubrication is essential.

The impact of viscous dissipation in natural convection was first investigated by Ref. [1], who noted that its influence becomes significant when the kinetic energy generated exceeds the amount of heat transferred. Ref. [2] further explored the inter-

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play between free and forced convection in a vertical channel with parallel plates, incorporating the effects of viscous dissipation. Utilizing a perturbation series method, he analyzed how viscous dissipation and buoyancy forces affect fluid flow in vertical channels. His findings indicated that for upward flows, both dimensionless velocity and temperature increase as the viscous dissipation parameter rises, whereas for downward flows, an increase in viscous dissipation leads to a decrease in velocity alongside an increase in temperature. Ref. [3] investigated the influence of viscous dissipation on laminar mixed convection within a vertical channel, revealing that viscous dissipation considerably impacts the temperature distribution, resulting in elevated temperatures and modified velocity profiles. In a subsequent study, Ref. [4] examined fully developed laminar mixed convection in an inclined channel and found that viscous dissipation enhances buoyancy effects and vice versa. Ref. [5] explored the role of viscous dissipation in mixed convection flow over a stretching sheet, demonstrating that it significantly affects both flow and temperature profiles, particularly at high Eckert numbers. Ref. [6] investigated natural convection flow between vertical parallel plates with time-periodic boundary conditions, observing that viscous dissipation heating within the channel raises fluid temperatures above the plate temperature when the Prandtl number is low. Ref. [7] conducted a theoretical study on the effects of boundary plate thickness and viscous dissipation on steady natural convection flow of an incompressible viscous fluid with variable viscosity, discovering that the volume flow rate increases with the Brinkman number while decreasing with greater boundary plate thickness. Ref. [8] analyzed stationary mixed convection flow with viscous dissipation in a horizontal channel characterized by adiabatic boundary walls and parallel velocity fields, noting that viscous dissipation led to a loss of symmetry in the velocity profile. Ref. [9] examined the impact of viscous dissipation on Alumina-water nanofluid transport in an asymmetrically heated microchannel, finding that it shifted the highest concentration and lowest fluid temperature away from the adiabatic wall while redistributing thermal conductivity variations. Ref. [10] studied thermal radiation and suction or injection effects alongside viscous dissipation on MHD boundary layer flow past a vertical porous stretched sheet, concluding that increased injection heat source parameters reduce fluid temperature for both suction scenarios, while rising radiation parameters decrease skin friction and heat transfer rates. Additionally, Ref. [11] assessed how amplitude and oscillating frequency affect heat transfer and electromagnetic-driven fluid flow along a horizontal circular cylinder, noting significant changes in temperature profiles due to viscous dissipation and magnetic fields, with temperatures rising alongside increased viscous dissipation.

Suction and injection refer to the transverse movement of fluid across a boundary surface. Suction involves extracting fluid from the system, while injection entails introducing fluid into the system simultaneously. The process of suction and injection significantly influences the flow field, thereby affecting the rate of heat transfer from the boundary surface. Additionally, it alters the velocity profile and modifies the shape of the boundary layer. These mechanisms are crucial for controlling flow in horizontal porous channels and have practical applications in various fields, including aerodynamics, space science, and environmental engineering, such as in film cooling and boundary layer management. Furthermore, suction and injection can be employed to delay the transition from laminar to turbulent flow. Ref. [12] investigated flow control on subsonic airfoils through suction and injection, concluding that suction enhances the lift coefficient, while injection decreases surface skin friction, ultimately reducing energy consumption during subsonic aircraft flight.

In general, skin friction tends to increase both the skin friction coefficient and the heat transfer coefficient, while injection has the opposite effect. Ref. [13] examined the flow and heat transfer characteristics of an incompressible Ostwald de Waele power-law fluid over an infinite porous plate subjected to suction and injection, finding that these processes reduce the temperature distribution. Ref. [14] explored the impact of suction and injection on unsteady hydromagnetic natural convection flow of a viscous reactive fluid between two vertical porous plates, taking thermal diffusion into account. Ref. [15] investigated the mass and heat transfer of Ag-kerosene oil nanofluid flowing over a cone under the influences of suction/injection, magnetic fields, thermophoresis, Brownian diffusion, and Ohmic-viscous dissipation. Their findings indicated that the heat transfer rate was greater for nanofluid flow over a cone compared to that over a wedge.

Understanding the effects of steady pressure-driven flow in horizontal porous channels is essential for a variety of engineering and environmental applications, such as geothermal energy extraction, groundwater management, and enhanced oil recovery. Ref. [16] conducted a study on single-phase forced convection of water and methanol through microchannels with rectangular cross-sections. Ref. [17] explored the convective heat transfer characteristics of nanofluids composed of γ-Al_2O_3 nanoparticles mixed with deionized water, finding a significant enhancement in convective heat transfer when using these nanofluids. Ref. [18] numerically investigated the heat transfer behavior of phase change material fluids under laminar flow conditions in both circular and rectangular microchannels. Ref. [19] analyzed fully developed laminar flow of cross fluids between parallel plates subjected to uniform heat flux.

Ref. [20] studied the effects of flow-induced vibrations on forced convection heat transfer from two tandem circular cylinders in laminar flow, discovering that wake interference can amplify the flow-induced vibrations of the downstream cylinder by as much as 81.32%, resulting in a maximum heat transfer enhancement of 28.3%.

This work studies the effect of viscous dissipation on steady -pressure driven flow in a horizontal channel. The effect of viscous dissipation on steady pressure-driven flow in a horizontal porous channel is highly relevant across multiple engineering and environmental applications. Understanding this phenomenon is essential for optimizing processes such as geothermal energy extraction, groundwater management, and enhanced oil recovery. Viscous dissipation plays a critical role in altering temperature distributions and influencing heat transfer rates within the fluid, which can significantly impact the efficiency of these applications.

Research has shown that viscous dissipation acts as an energy source that modifies the thermal and flow characteristics of fluids in porous media. For instance, studies indicate that as viscous dissipation increases, it can enhance heat transfer rates, thereby improving system performance. This is particularly important in scenarios where maintaining optimal temperature conditions is crucial for operational efficiency.

Furthermore, the dynamics of viscous dissipation can lead to changes in velocity profiles and boundary layer behavior, which are vital for ensuring effective fluid transport in porous channels. The implications of these changes extend to various industries, including petroleum engineering, where understanding flow behavior under different thermal conditions can lead to more effective extraction methods. Because of the nonlinearity of the energy equations involving viscous dissipation effects, many researches have ignored this very important phenomenon, thereby rendering the outcome of such research quantitatively inadequate. This work therefore, is to address the shortcomings due to ignoring the viscous dissipation effects in fluid flow problems. The impact of viscous dissipation on steady pressure-driven flow in horizontal porous channels is significant for advancing engineering practices and enhancing the performance of systems reliant on fluid movement through horizontal channel. The insights gained from studying this effect will contribute to improved designs and operational strategies across a wide range of applications.

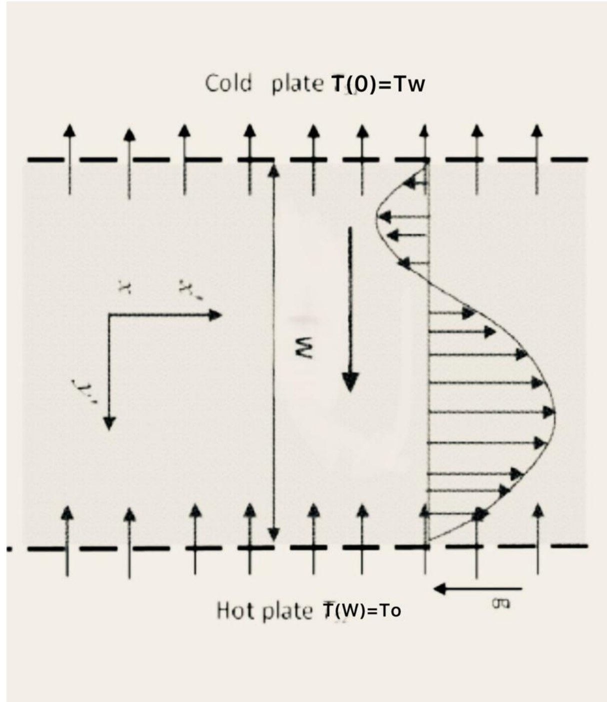

Figure 1. Schematic diagram of the flow [21].

2. MATHEMATICAL ANALYSIS

Considering the laminar flow of a viscous incompressible fluid within a horizontal channel. The x'-axis lies on the axial plane and this shows the direction of the flow. The y'-axis lies along the perpendicular to the walls. The flow is fully developed and in the steady- state. The flow is steady, and pressure driven in the horizontal porous channel with particle suction/injection. The viscous dissipation effect is also taken into consideration. At every instance, the flow is subjected to suction of fluid through any of the porous boundary plates with equal rate of injection through the opposite plates. The channel walls are infinite in the x' direction which makes the flow becomes one dimensional, hence velocity is a function of y' only.

Considering the equation of continuity $\nabla \cdot U = 0$ gives:

$$\frac{du'}{\partial x'} = 0.$$

Component U depends on the only y-coordinate. Considering the importance of conservation of mass in convective flow, the rate at which fluid is sucked out of the channel must be equal to the rate at which the fluid is injected into the flow channel.

The governing hydrodynamic and thermodynamic equations in dimensional form are:

$$\nu\left(\frac{d^2u'}{dy'^2}\right) - v_0\frac{du'}{dy'} + \frac{1}{\rho}\frac{dp'}{dx} = 0, \quad (1)$$

$$\alpha\frac{d^2T'}{dy'^2} - v_0\frac{dT'}{dy'} + \frac{v_0}{C_p}\left(\frac{du'}{dy'}\right)^2 = 0, \quad (2)$$

subject to

$$u'^{(0)} = 0,\ u'(h) = 0,\ T(0) = T_w,\ T(h) = T_0, \quad (3)$$

as displayed in Figure 1.

The dimensional variables are defined as:

$$u = \frac{u'}{u_0}, \theta = \frac{T' - T_0}{T_w - T_0}, y = \frac{y'}{h}, x = \frac{x'}{h}. \quad (4)$$

Utilizing the dimensional variable in equation (4) into equations (1) and (2) yields:

$$\frac{d^2u}{dy^2} - S\frac{du}{dy} = -A, \quad (5)$$

$$\frac{d^2\theta}{dy^2} - S\,\mathrm{Pr}\frac{d\theta}{dy} + Br\left(\frac{du}{dy}\right)^2 = 0, \quad (6)$$

$$u(0) = 0, u(0) = 0, \theta(0) = 1, \theta(1) = 0, \quad (7)$$

where $\mathrm{Pr} = \frac{\nu}{\alpha}$ is the Prandtl number, $S = \frac{v_0 h}{\nu}$ is the suction/injection parameter, $Br = EcPr$ is the Brinkman number, and $A = \frac{h}{\rho u_0 \nu}\frac{dp'}{dx'}$ is the pressure gradient parameter.

2.1. METHOD OF SOLUTION

The dimensionless equations in equations (5) and (6) along with the boundary conditions in equation (7) are solve by the method of determined coefficient. The exact solution for the dimensionless temperature $\theta(y)$ and the dimensionless velocity $u(y)$ can be expressed as:

$$u(y) = C_1 + C_2 e^{sy} + \frac{A}{S}y, \quad (8)$$

$$\theta(y) = C_3 + C_4 e^{s\,\mathrm{Pr}\,y} + k_1 e^{m_1 y} + k_2 e^{m_2 y} + k_3 y, \quad (9)$$

where

$$C_1 = \frac{A}{S(1 - e^s)}, C_2 = -C_1, \quad (10)$$

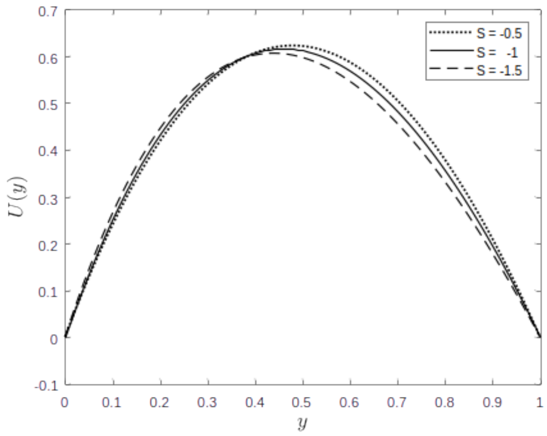

Figure 2. Velocity profile for different suction values.

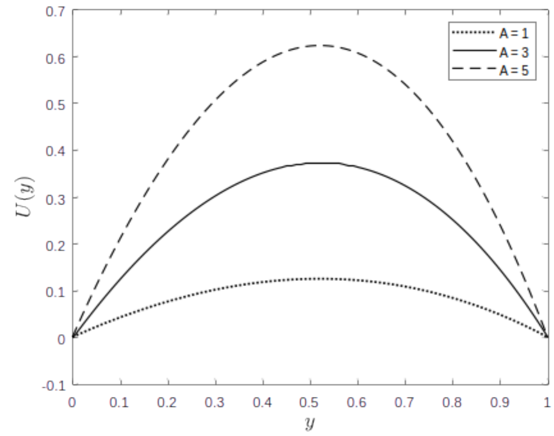

Figure 3. Velocity profile for different pressure gradient.

$$L_1 = (SC_2)^2\,, L_2 = 2AC_2, L_3 = \frac{A^2}{S^2}, \tag{11}$$

$$k_1 = \frac{-BrL_1}{{m_1}^2 - S\,\mathrm{Pr}\,m_1}, k_2 = \frac{-BrL_2}{{m_2}^2 - S\,\mathrm{Pr}\,m_2}, k_3 = (BrL_3/SPr)\,, \tag{12}$$

$$C_3 = 1 - (k_1 + k_2 + C_4)\,, \tag{13}$$

$$C_4 = \frac{1 + (e^{m_1} - 1)\,k_1 + k_2\left(e^{m2} - 1\right) + k_3}{1 - e^{sPr}}, \tag{14}$$

3. RESULTS AND DISCUSSION

The steady-pressure driven flow of an incompressible viscous fluid in a horizontal porous channel where one of the channel walls is heated isothermally. The basic parameters that govern the laminar flow of the system under consideration include Prandtl number (Pr) that measures the thermal diffusivity of the fluid, Brinkman number (Br) that measures the viscous dissipation and Suction/Injection parameter was discussed. For S >0 which is suction on the cold wall ($y = 1$) and S<0 a simultaneous injection on the hot wall($y = 0$). The results were presented by the use of line graphs in Figure 2.

Figure 2 is a display of the variation of Velocity with Suction, it is noted that for S<0 (Injection) on the hot wall y=0 and there exist a corresponding suction when S>0 on the cold wall increasing the S>0 mean sucking out cold air and there exist an injecting of the applied heat into the system leading to an enhance convective flow current leading to increase in the fluid flow with an increase in Injection at the wall $y = 0$, however it is seen that the Suction S affects the symmetric nature about the midchannel, there is a distortion to the hot wall of the channel.

In Figure 3, Velocity profile for different values of Pressure gradient was presented, it was observed that increase in pressure gradient means an increased in the driving force of the velocity fluid leading to higher convective current which result to increase in velocity as the pressure gradient increases.

Figure 4 depicts the variation of temperature profile versus pressure gradient, It was observed that a temperature increases with increasing value of pressure gradient. This is attributed to the fact that as the fluid motion driving force increases this will leads to rapid entropy change leading to increase in heat transfer within the channel.

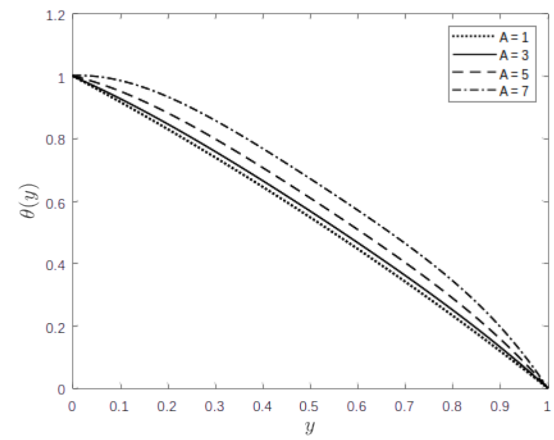

Figure 4. Temperature profile for different pressure gradient.

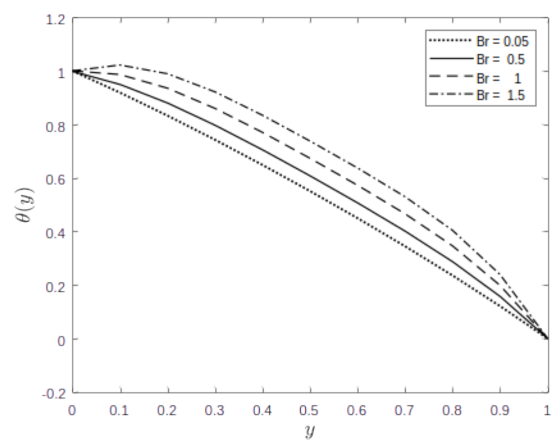

Figure 5. Variation of temperature with different Brinkman number.

Figure 5 shows the variation of temperature with different Brinkman number, it was seen that as the Brinkman number in-

Table 1. Skin friction on the surface of the channel plates.

A	τ_0				τ_1			
Br	S = -1	S = -0.5	S = 0.5	S = 1	S = -1	S = -0.5	S = 0.5	S = 1
0.1	0.0582	0.0541	0.0459	0.0418	-0.0418	-0. 0459	-0. 0541	-0. 0582
0.5	0.2910	0.2707	0.2293	0.2090	-0. 2090	-0. 2293	-0. 2707	-0.2910
1.0	0.5820	0.5415	0.4585	0.4180	-0. 4180	-0. 4585	-0. 5415	-0.5820

Table 2. Nusselt number on the surface of the channel plates, when Pr =0.72, *Br*=0.05.

A	Nu_0				Nu_1			
Br	S = -1	S = -0.5	S = 0.5	S = 1	S = -1	S = -0.5	S = 0.5	S = 1
0.1	-1.4031	-1.1918	-0.8317	-0.6830	-0.6826	-0.8299	-1.1897	-1.4026
0.5	-1.4092	-1.2167	-0.8538	-0.6879	-0.6777	-0.8077	-1.1649	-1.3965
1.0	-1.4283	-1.2945	-0.9229	-0.7032	-0.6624	-0.7386	-1.0871	-1.3773

Table 3. Nusselt number on the surface of the channel plates, when Pr =7.0, *Br*=0.05.

A	Nu_0				Nu_1			
Br	S = -1	S = -0.5	S = 0.5	S = 1	S = -1	S = -0.5	S = 0.5	S = 1
0.1	-7.0068	-3.6104	-0.1095	-0.0065	-0.0063	-0.1085	-3.6299	-7.0060
0.5	-7.0163	-3.6455	-0.1214	-0.0080	-0.0048	-0.0966	-3.5724	-6.9965
1.0	-7. 0460	-7.7805	-0.1586	-0.0127	-0.0001	-0.0594	-3.4627	-6.9668

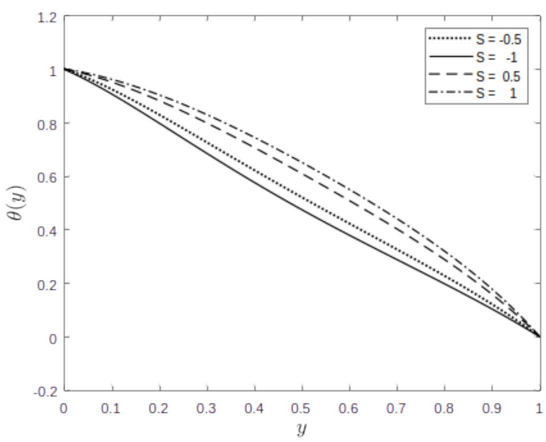

Figure 6. Temperature profile versus different suction.

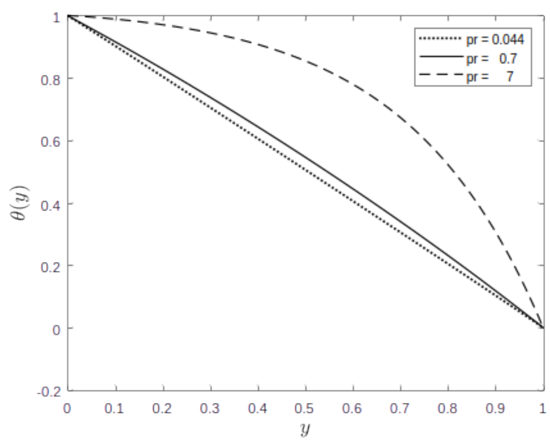

Figure 7. Temperature profile for different Prandtl number.

creases, which is increasing heating within the channel an increased in the temperature profile was observed, this is as a result of accumulation of heat energy within the channel in addition to the injection of the applied heat at the wall $y = 0$ into the channel. There is also observed a high temperature gradient.

Figure 6 reports the plot of temperature within the channel with respect to Suction for S>0, it is noticed that a growing the Suction at the channel wall $y = 1$, increase in temperature was observed, this is as a result of cold air sucked out of the channel and in addition with generation of heat within the system as a result of the viscous dissipation in the channel an accumulation of heat will be in the system. However, as injection at the wall $y = 0$ increases there is a decrease in temperature.

Figure 7 shows the variation of temperature with different Prandtl number values, it is seen that temperature increases as the Prandtl number increase, this is as a result of the combined effect of the injection of the applied heat at the wall $y = 0$ and the generation of heat within the system as a result of viscous dissipation. However, this is seen to suppress the effect of increasing of Prandtl number which normally leads to low thermal diffusivity within the fluid.

The skin friction and rate of heat transfer on the surface of the horizontal parallel plates are computed.

3.1. SKIN FRICTION

To compute the shear stress (τ) on the surface of the channel plates,

$$\tau_0 = \left.\frac{du}{dy}\right|_{y=0} = SC_2 + \frac{A}{S}, \tag{15}$$

$$\tau_1 = \left.\frac{du}{dy}\right|_{y=1} = SC_2\, e^s + \frac{A}{S}, \tag{16}$$

while the rate of heat transfer (Nu) on the surface of the channel plates is computed via:

$$Nu_0 = \left.\frac{d\theta}{dy}\right|_{y=0} = S\,\mathrm{Pr}\,C_4 + m_1 k_1\; m_2, \tag{17}$$

$$Nu_1 = \left.\frac{d\theta}{dy}\right|_{y=1} = S\,\mathrm{Pr}\,C_4 e^{SPr} + m_1 k_1 e^{m_1} + m_2 k_1 e^{m_2}. \tag{18}$$

In Table 1, it is observed that has Suction increases, the skin friction decreases, this will be as a result of removal of fluid near the wall thereby reducing the velocity gradient. However, increasing the Brinkman number increases the skin friction, this is as a result of increase in viscous dissipation which leads to increase in the convective current leading to increase in the velocity gradient at the wall.

In Table 2, the effect of suction on the Nusselt number was displayed, this was observed increasing Suction leads to an increased in Nusselt number, this is as a result of increased velocity gradient which will invariably leads to increasing high convective heat transfer.

Table 3 helps to shows the effect of different Prandtl number on Nusselt number, comparing the result in Table 2 and Table 3, it was discovered that the fluid having a higher Prandtl number of 7.0 has lower Nusselt number compared to the fluid with lower Prandtl number of 0.71 as seen in Table 2. This is because fluid with higher Prandtl number means fluid with low heat diffusivity that is heat transfer to the plate is low. Therefore, leading to low Nusselt number as seen in the values in Table 3.

4. CONCLUSION

In conclusion, this study highlights the significant impact of viscous dissipation on steady pressure-driven flow in a horizontal porous channel. The results demonstrate that viscous dissipation leads to an increase in the fluid temperature within the channel, which directly influences the thermal dynamics of the flow. Additionally, the velocity of the fluid was found to increase as the pressure gradient is raised, indicating a strong dependency of flow behaviour on the applied pressure difference. Furthermore, the investigation reveals that when suction is applied through the cold plate, the heat transfer rate on the cold plate increases, while it decreases on the heated plate. These findings provide valuable insights into the complex interplay between thermal and flow characteristics in porous media, suggesting potential applications in systems where thermal management is crucial. The observed behavior emphasizes the importance of considering viscous dissipation and its effects on both heat transfer and fluid dynamics when designing and analyzing such systems.

DATA AVAILABILITY

We do not have any research data outside the submitted manuscript file.

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