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Innovative mathematical application of game theory in solving healthcare allocation problem

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ABSTRACT

Game theory being a mathematical framework for analyzing strategic interactions among decision makers is increasingly applied to complex problems in healthcare resource allocation. This innovative research strategically applied the game theory model to solve the problem of healthcare allocation. It determined the strategies of Player I (Company A) and Player II (Competitor- Company B), established those strategies using QM (Quality Management) software for Windows, determined the amount of gain or loss experienced by Player I and Player II during the allocations, created various game theory plots, such as the Row's graph against Column's strategy and the Column's graph against Row's strategies for a pure strategy, and produced various game theory plots, such as the Row's graph for mixed strategy with given expected loss (%). For it to profit by #1,000,000 (for example) in the case that Company B expands to X location, company A must assign the patients' bed production plant to location (strategy) D. If Company B introduces the 50% discount or makes #7,000,000, then company A does not have to assign any amount of vaccines manufactured in their firm. The objective is to stop outbreaks in high-risk locations and reduce the spread of infectious diseases. Consequently, a mixed strategy is used, with the aim of maximizing the overall quality of care given to patients while making sure that each facility has the resources necessary for efficient operation.

Keywords: Game theory, Healthcare, Mathematical application and resources allocation.

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1. INTRODUCTION

Game theory, a mathematical framework for analyzing strategic decision-making, has diverse applications, including healthcare resource allocation. It helps in understanding the approaches involved in playing a strategic game. Strategic games refer to situations where players must make decisions based on the choices and actions of others, and where the outcomes depend on the strategies adopted by each player. These games involve strategy, analysis, and decision-making to achieve the best possible outcome for the player.

In the context of the healthcare system, strategic games refer to situations where different healthcare providers or entities must make decisions based on the actions of others, such as patients, insurers, or other providers. These games involve strategic decision-making and analysis to optimize resource allocation, quality of care, cost-effectiveness, and overall performance

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within the healthcare system. In line with Karlsson (2021) [1], game theory is the study of decision-making under conflicting interests.

The concept of the interdependence of actions of players in game theory refers to the idea that the decisions and choices made by each player in a game can influence and are influenced by the decisions of other players. In other words, the outcome of the game depends not only on an individual player's actions but also on how those actions interact with the actions of others. Game theory according to Nowak and Sigmund (2004) [2] is seen as the appropriate tool whenever the success of an individual depends on others.

This interdependence [3, 4] creates a complex network of strategic interactions, as players must anticipate and respond to the decisions of their opponents to achieve their own goals. The game involves two types of strategic influence - one where players take turns and consider each other's actions (sequential interdependence) and another where players make decisions simultaneously without considering others' actions (simultaneous interdependence).

Understanding and managing this interdependence is central to game theory, as players must consider not only their own optimal strategies, but also how those strategies will be affected by the choices (Askari *et al*. 2019) [5] of others. This can lead to scenarios where players must balance cooperation and competition, analyze potential outcomes, and adapt their strategies in response to changing circumstances to maximize their chances of success.

Game theory can be applied to the healthcare system to study the interdependence of actions of various players such as patients, providers, insurers, and policymakers. Each player's actions can have a direct impact on the outcomes of the healthcare system, leading to an intricate web of relationships and dependencies.

Game theory is widely used in healthcare to analyze strategic interactions among hospitals, insurers, policymakers, and patients. As healthcare systems grow complex, stakeholders must make critical decisions about resource allocation, pricing, and public health policies. Game theory helps optimize outcomes by balancing competition, efficiency, and fairness. Models like Nash Equilibrium and Prisoner's Dilemma explain hospital competition, doctor-patient interactions, and policy compliance. Hospital pricing resulting in unregulated competition can increase costs. Medical technology investments in hospitals result in overinvest staying competitive. Doctor-patient decision-making results in showing how shared decision-making improves treatment adherence. Health insurance markets demonstrate how insurers adjust premiums to attract enrollees. In public health, epidemic control strategies use game theory to model compliance with social distancing and vaccination policies. These applications help policymakers design strategies that enhance healthcare efficiency and equity.

Some authors such as Okeke (2020) [6], Okeke & Akpan (2019) [7], and Okeke & Ifeoma (2024, 2023) ([8], [9]) identified varieties of modelling leading to physical applications and sensitivity of coronavirus disparities in Nigeria. Theorems such as fixed point had also been identified [10]. Some numerical stability of flows in physical application was identified in Okeke & Peters (2019) [11]. The principle of maximum was used to prove some common uniqueness of pair in a metric space of system of linear Volterra integral equations of the second order. Ref. [12] identified varieties of modelling leading to healthcare patients' applications. Accordingly, Okeke, et al. (2019) [13] identified HIV infection modelling.

The aim of this study is to strategically apply the model of Game Theory to Healthcare Allocation Problem. The objectives of this study include: determining the Strategies of Player I and Player II (Competitor), establishing the Strategies of Player I and Player II using QM (Quality Management) software for Windows, finding out how much gain or loss of the Player I and Player II during the allocations, creating various plots for the Game theory such as the Row's graphs against Column's strategy and Column's graph against Row's Strategies for a pure strategy, and producing various plots for the Game theory such the Row's graph for mixed strategy with given expected loss (%). QM software simplifies complex quantitative methods, offering an intuitive interface that reduces the need for coding and manual calculations. It enhances efficiency and accuracy by automating operations research and management science problems. Its versatility across optimization, forecasting, and inventory management makes it superior to traditional mathematical tools.

2. REVIEW OF RELATED LITERATURES

Game theory is a branch of mathematics that studies strategic interactions between rational decision-makers. It is widely used in various fields such as economics, political science, biology, and computer science to analyze and predict the behavior of individuals and groups in competitive situations.

One of the foundational concepts in game theory is the Nash equilibrium (1951) [14], named after Nobel laureate John Nash, which is a state in a game where each player's strategy is optimal given the strategies of the other players. This equilibrium helps in understanding and predicting the outcomes of strategic interactions. Hospitals compete for limited resources like staff, funding, and equipment, leading to strategic decision-making influenced by Nash Equilibrium. In an Intensive Care Unit (ICU) bed allocation, hospitals set admission criteria based on competitor actions to avoid overwhelming capacity. Hiring specialized staff leads to stable salary offerings, preventing excessive wage inflation. Investments in advanced medical technology are balanced to maintain competitiveness without unnecessary financial strain. Hospitals adjust pricing strategies to attract insured patients while maintaining profitability. Geographic expansion decisions are made strategically to optimize patient reach without market oversaturation.

Another important concept in game theory is the Prisoner's dilemma identified by Kuhn (2024) [15], a classic example of a game where individuals acting in their own self-interest do not produce the best outcome for the group as a whole. This dilemma illustrates the idea of cooperation and the benefits of communication and coordination among players. Game theory has been applied in various real-world scenarios, such as auctions, bargaining, and negotiation, to analyze and optimize strategies. It also has implications for understanding evolutionary dynamics (Sigmund & Nowak, 2004) [16], social behavior, and decision-making in uncertain environments.

One of the seminal works done by von Neumann & Morgenstern (1944) [17] on game theory is the theory of games and economic behavior. Their work laid the foundation for the formal study of strategic interactions and has had a lasting impact on various disciplines. Overall, game theory provides a powerful framework for analyzing decision-making in competitive situations and has become an essential tool in studying social, economic, and biological systems. By applying game theory principles, researchers can better understand the dynamics of strategic interactions and make informed predictions about behavior in complex environments [18].

Moving forward, game theory has emerged as a powerful analytical tool in various fields, including healthcare. Central concepts like the Nash Equilibrium and the Prisoner's Dilemma identified in Tucker (1950) [19] have provided insights into decision-making under strategic interaction. The Nash Equilibrium describes a situation in which no player has anything to gain by changing their strategy unilaterally. This concept has been applied to healthcare markets (Pauly, 1968) [20], healthcare insurance design, and provider competition. Researchers have shown how Nash Equilibria can help explain the strategic interactions between hospitals, insurers, and patients. In response to every possible emergency scenario, Fargetta *et al*. (2022) [21] examined a pre-event policy wherein healthcare institutions aim to minimize the upfront transportation time and the cost of purchasing medical supplies, as well as a recourse decision process that optimizes the expected overall costs and the penalty for the previous plan.

An equilibrium predicts which path will be followed in each instance by predicting how each participant will move; this prediction is known as the equilibrium path (Jackson, 2011) [22]. A state known as the Nash equilibrium is one in which no individual variation will result in improvement. The concept of Nash equilibrium is crucial to game theory. It alludes to the presumption that the game is being played by n participants. Each player chooses the best course of action to maximize his or her own interests based on the plans of others. In other words, no one has a compelling enough incentive to upset this balance given the tactics of others (Gao & Yao, 2018)[23]. Recent trends like specialization, the spread of high technology, concentration and privatization tendencies, and growing competitive pressure all contribute to healthcare economization, which presents serious ethical issues, such as the fact that patients and doctors are not equally strong partners in the sense of a symmetric supply and demand relationship (Rogowski & Lange, 2022)[24]. According to Zhou *et al*. (2014) [25] in Sun *et al*. (2017) [26], health policy makers and health systems should prioritize equality and efficiency in the distribution of health resources and the use of health services.

3. RESEARCH METHODOLOGY

This section discusses the methodology of mathematical game theory that is use to solve healthcare problems.

3.1. METHODOLOGY OF MATHEMATICAL GAME THEORY

The graphical method steps (Rule) are used in games with no saddle point and having a pay-off matrix of type $n\times 2$ or $2\times n$. For Saddle point testing, we apply the maximin (minimax) principle to analyze the game.

Table 1. The payoff summary for case 1.

Expansion payoff (# 000,000)	Player II (Company B)	
Player I (Company A)	X	Y
C	-3	7
D	1	2

PROCEDURES

- Select maximum from the minimum of rows; $\max_j\{\min_i\{v_{ij}\}\}$.
- Select minimum from the maximum of columns; $\min_i\{\max_j\{v_{ij}\}\}$.
- For saddle point, $\max_j\{\min_i\{v_{ij}\}\} = \min_i\{\max_j\{v_{ij}\}\}$.
- For no saddle point, $\max_j\{\min_i\{v_{ij}\}\} \neq \min_i\{\max_j\{v_{ij}\}\}$.

The minimax principle ensures fairness in healthcare allocation by minimizing the maximum disadvantage and prioritizing the worst-off populations, aligning with ethical principles of justice and equity. It provides a robust framework for decision-making in uncertain healthcare environments by addressing critical needs first. By preventing extreme disparities, the minimax approach optimizes overall health outcomes and promotes equitable resource distribution. Following the Minimax Theorem in Ferguson (2020) [27] proven by Owen (1967) [28], we say that every finite game has a value, and both players have minimax strategies.

4. RESULTS AND FINDINGS

The cases of how game theory are applied in solving healthcare allocation problem setting.

Case 1: Given an orthopedic company A that produces different kinds of beds (example; Critical Care, Surgical, Maternity) which will be made available to certain patients. An operational officer (PO) in the company A is planning to expand the company's operations in certain different locations either location C or location D. He learned that their toughest competitor, company B is planning to expand to the same state either in X or Y locations. If the orthopedic company A expands to C, it will loss #3,000,000 if company B expands X or gain #7,000,000 if B expands to Y. If A expands to D, it will gain #1,000,000 if B expands to X or gain #2,000,000 if B expands to Y. The payoff summary is presented in the Table 1. Table 2 shows the game theory result from the analytic calculations for the Case 1. Table 3 shows game theory result from the QM Software Window for the Case 1.

A payoff is a mapping to the real numbers that depends on the other players' decisions.

Analytic value (Row):

$$\max_j\left\{\min_i\left(v_{ij}\right)\right\} = 1.$$

Table 2. Game theory result from the analytic calculations for the case 1 (final strategy).

Expansion payoff (# 000,000)	Player II (Company B)
Player I (Company A) with Strategy D	X
D	1
Strategy	D

Table 3. Game theory result from the qm software window for the case 1.

	X	Y	Row Mix
C	-3	7	0
D	1	2	1
Column Mix −− >	1	0	
Value of game (to row)	1		

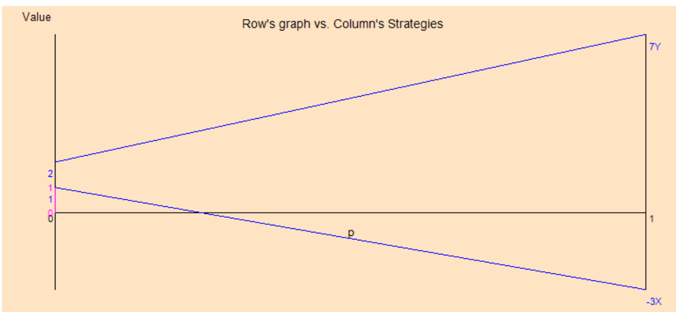

Figure 1. Row's graph against column's strategies for the game theory result from the QM software window for the case 1.

Since:

$$\min\{-3, 7\} = -3, \quad \min\{1, 2\} = 1.$$

Therefore,

$$\max\{-3, 1\} = 1.$$

Analytic value (Column):

$$\min_i \left\{ \max_j \left(v_{ij} \right) \right\} = 1.$$

Since:

$$\max\{-3, 1\} = 1, \quad \max\{7, 2\} = 7.$$

Therefore,

$$\min\{1, 7\} = 1.$$

Case 2: Consider a pharmaceutical company A that produces vaccines. The healthcare system through their executive marketing planner needs to decide how to allocate a limited supply of vaccines among different regions or populations. She plans to promote more subscriptions through price discounts either 30% off or 20% off. She also learned that their closest competitor, company B is planning to promote more subscriptions through discount either 50% off or 25% off. If the company A launches 30% off, it will gain nothing. If company B launches the 50% off or gain #7,000,000 if company B launches the 25% off. If company A launches the 20% off, it will lose #2,000,000 if company B launches the 50% off or lose #5,000,000 if company B launches the 25% off. The payoff summary is presented in Table 4. Table 5 shows game theory result from the analytic calculations for the Case 2. Table 6 shows game theory result from the QM Software Window for the Case 2.

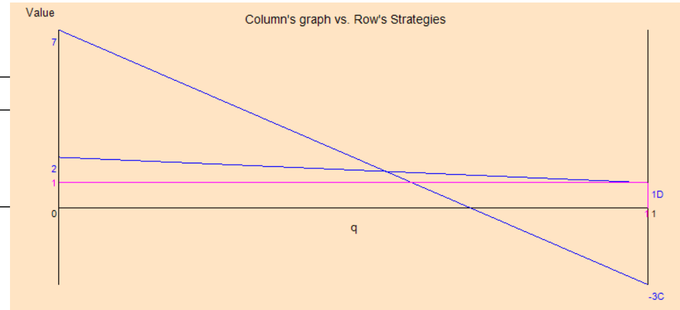

Figure 2. Column's graph against row's strategies for the game theory result from the QM software window for the case 1.

Table 4. The payoff summary for case 2.

Expansion payoff (# 000,000)	Player II (Company B)	
Player I (Company A)	50%	25%
30% off	0	7
20% off	-2	-5

Analytic value (Row):

$$\max_j \left\{ \min_i \left(v_{ij} \right) \right\} = 0.$$

Since:

$$\min\{0, 7\} = 0, \quad \min\{-2, -5\} = -5.$$

Therefore,

$$\max\{0, -5\} = 0.$$

Analytic value (Column):

$$\min_i \left\{ \max_j \left(v_{ij} \right) \right\} = 0.$$

Since:

$$\max\{0, -2\} = 0, \quad \max\{7, -5\} = 7.$$

Therefore,

$$\min\{0, 7\} = 0.$$

Case 3: Consider a group of healthcare providers (G) who needs to decide how to allocate limited resources (e.g. medical equipment, staff, funding) among different healthcare facilities. A representative of the group (G) is planning to partner with foreign providers A or B. He learned that another competitor (C) closet to (G) is planning to partner with a foreign healthcare provider G_1 and G_2. If G partners with A, it will lose #5,000,000 if C partners with G_1 or gains #7,000,000 if C partners with G2. If G partners with B, it will gain #2,000,000 if C partners with G_1

Table 5. Game theory result from the analytic calculations for the case 2 (final strategy with 50% strategy in player II).

Expansion payoff (# 000,000)	Player II (Company B)	
Player I (Company A) with 30% Strategy		Strategy
A	0	50%
Strategy	30%	Pure

Table 6. Game theory result from the QM software window for the case 2.

	50%	25%	Row Mix
30%	0	7	1
20%	-2	-5	0
Column Mix −− >	1	0	
Value of game (to row)	0		

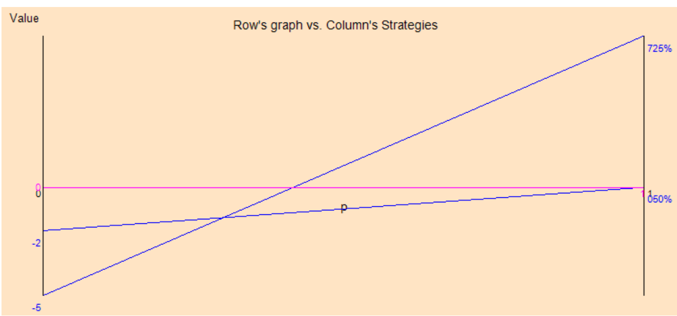

Figure 3. Row's graph against column's strategies for the game theory result from the QM software window for the case 2.

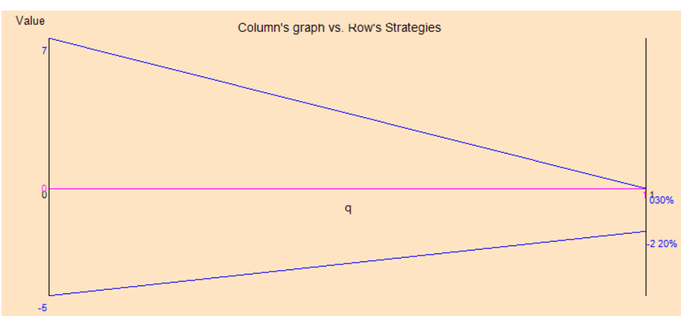

Figure 4. Column's graph against row's strategies for the game theory result from the QM software window for the case 2.

or loss #4,000,000 if C partners with G_2. The payoff summary is presented in the Table 7. Table 8 show game theory result from the QM Software Window for the Case 3. Table 9 shows game theory result showing the row's expected value for the Case 3. Table 10 shows game theory result showing the column's expected value for the Case 3. Table 10 shows game theory result showing the computations of Maximin and Minimax values for the Case 3.

Analytic value (Row):

$$\max_j \left\{ \min_i \left(v_{ij} \right) \right\} = -4.$$

Table 7. The payoff summary for case 3 given expected loss.

Expansion payoff (# 000,000)	Player II (Company C)		
Player I (Company G)	G1	G2	Expected Loss (%)
Company A	-5	7	-45%
Company B	2	-4	-45%

Table 8. Game theory result from the QM software window for the case 3.

	G1	G2	Expected Loss	Row Mix
A	-5	7	-.45	.35
B	2	-4	-.45	.65
Column Mix −− >	0	0	1	
Value of game (to row)	-.45			

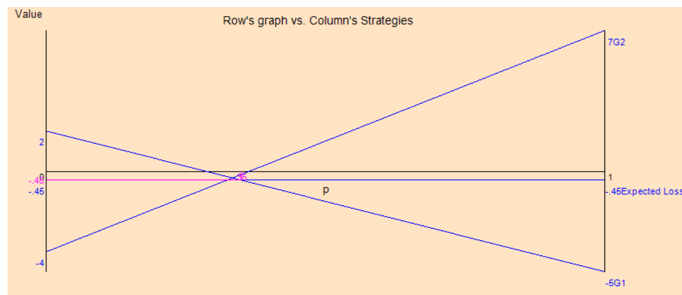

Figure 5. Row's graph against column's strategies for the game theory result from the QM software window for the case 3.

Table 9. Game theory showing the row's expected value for the case 3.

	Payoff 1*	Payoff 2*	Payoff 3*	Expected Value
Column's Optimal Mix	0	0	1	
A	0	0	-.45	-.45
B	0	0	-.45	-.45
Value of game (to row)				-.45

Since:

$$\min\{-5, 7, -0.45\} = -5, \quad \min\{2, -4, -0.45\} = -4.$$

Therefore,

$$\max\{-5, -4\} = -4.$$

Analytic value (Column):

$$\min_i \left\{ \max_j \left(v_{ij} \right) \right\} = -0.45.$$

Since:

$$\max\{-5, 2\} = 2, \quad \max\{7, -4\} = 7.$$

$$\max\{-0.45, -0.45\} = -0.45.$$

Therefore,

$$\min\{2, 7, -0.45\} = -0.45.$$

Table 10. Game theory showing the columns expected value for the case 3.

	Optimal Row*	G1	G2	Expected Loss
Row 1 mix*	.35	-1.75	2.45	-.16
Row 2 mix*	.65	1.3	-2.6	-.29
Expected Value(Col sum)		-.45	-.15	-.45
Value of game (to row)	-.45			

5. DISCUSSION OF THE RESULTS

This section discusses the results of the study. For Case 1, first, draw two lines -3 unit distance apart and make a scale on each. The two lines represent strategies of player II. If player I selects strategy C, Player II can win −3 or 7 units depending on X or Y selection of strategies of player II. The value −3 is plotted along vertical axis under strategy X and the value 7 is plotted along vertical axis under strategy Y. A straight line joining the two points is then drawn. Similarly, the strategies D can be plotted. Figures 3 - 5 follow subsequently.

The sketches provided a clear visual comparison of the outcomes for different strategies. The graphs help players recognize which strategies yield better payoffs. The graph can indicate points where neither player benefits from changing strategies. Straight lines between payoffs show possible benefits of mixing strategies. They convert payoff tables into an easy-to-understand visual format.

The company A needs to allocate the patients' bed production factory to location (strategy) D to enable them to make a gain of #1,000,000 if B expands to X location. For Case 2, the company A needs not to allocate any quantity of vaccines produced in their company if company B launches the 50% off or gain #7,000,000. The goal is to minimize the spread of contagious disease and prevent outbreaks in high-risk areas. For Case 3, the result is a mixed strategy. The goal is to maximize the overall quality of care provided to patients while ensuring that each facility has the resources it needs to function effectively. Mixed strategies involved assigning probabilities to a player's pure strategies, enabling random or probabilistic decisions. In the context of healthcare, when no clear optimal solution exists, the overall performance can improve by having healthcare providers select strategies randomly, based on a defined probability distribution. This approach allows for flexibility and adaptability in decision-making, which can enhance the effectiveness and efficiency of healthcare delivery when direct optimal strategies are not available.

6. CONCLUSION

This section draws conclusion from the study. The graphs and tables obtained from the QM software simplified data interpretation, helping decision-makers optimize resources, predict future trends, and improve efficiency. Understanding these outputs enables better strategic planning, minimizing risks and maximizing benefits in the various companies and operational contexts. Future advancements in QM software could integrate machine learning for adaptive and predictive analytics. Cloud-based solutions would enhance scalability by handling large datasets efficiently. Hybrid modeling approaches, including heuristics and neural networks, could improve problem-solving accuracy. Enhanced user interfaces with interactive dashboards and customization would make the software more user-friendly. Integrating QM software with Big Data and IoT could enable real-time decision-making in industries like healthcare and supply chain management. QM software relies on accurate and complete data; errors or missing values can lead to poor decision-making. Handling large datasets is challenging as most QM tools are designed for smaller-scale problems. Many models assume static data, making them less effective in dynamic, real-time decision-making. Despite being user-friendly, QM software requires a basic understanding of quantitative methods, posing a barrier for non-experts. Implementing QM software at an enterprise level may require additional resources and training. Many organizations face cost constraints when adopting advanced QM tools. Addressing these challenges can improve the efficiency and applicability of QM software in diverse fields. Game theory and QM software can transform healthcare by optimizing resource allocation and improving patient outcomes. By modeling stakeholder interactions, game theory helps design fair and efficient healthcare policies. QM software enhances decision-making by analyzing scenarios, predicting outcomes, and managing uncertainty. As healthcare becomes more data-driven, integrating game theory with QM tools ensures equitable and effective strategies. Advancements in big data, and real-time analytics will further enhance adaptive healthcare decision-making.

DATA AVAILABILITY

The datasets utilized and analyzed in this study are available from the corresponding author upon reasonable request.

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